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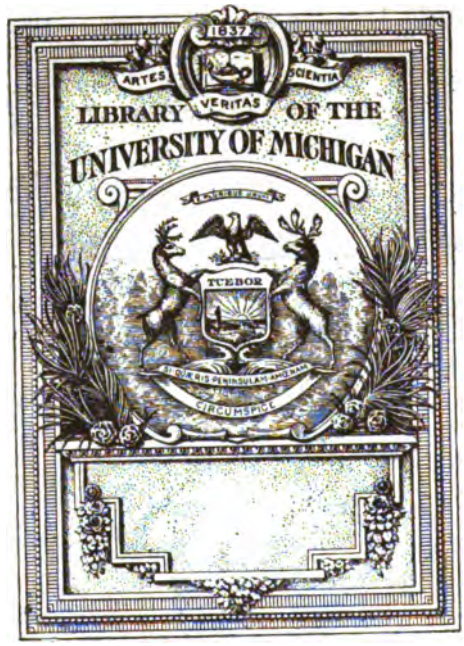
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THEORY

OF

INTEREST,

SIMPLE AND COMPOUND,

DERIVED FROM FIRST PRINCIPLES,

AND

APPLIED TO ANNUITIES OF ALL DESCRIPTIONS.

THEORY

OF

INTEREST,

SIMPLE AND COMPOUND,

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AND

APPLIED TO ANNUITIES OF ALL DESCRIPTIONS.

CONTAINING,

- Arithmetical and Geometrical Progreſſions.
- Simple and Compound Intereſt, with Tables.
- Annuities at Simple and Compound Intereſt, with Tables.
- Of a Sinking Fund to extinguiſh the National Debt.
- The Value of a Perpetuity.
- Annuities on Lives, with Tables.
- M. De Moivre's Hypotheſis of Equal Decrements, with 22 Problems, ſhewing the Value of Life in all its Varieties.
- Inſurance on Lives.
- Of the Expectation of Life.
- Of the Britiſh State Lottery.
- Of Tontines, and Inſurance from Fire, &c.

WITH

AN ILLUSTRATION OF THE WIDOWS SCHEME IN THE CHURCH OF SCOTLAND.

By the Reverend Mr DAVID WILKIE,
MINISTER OF CULTS.

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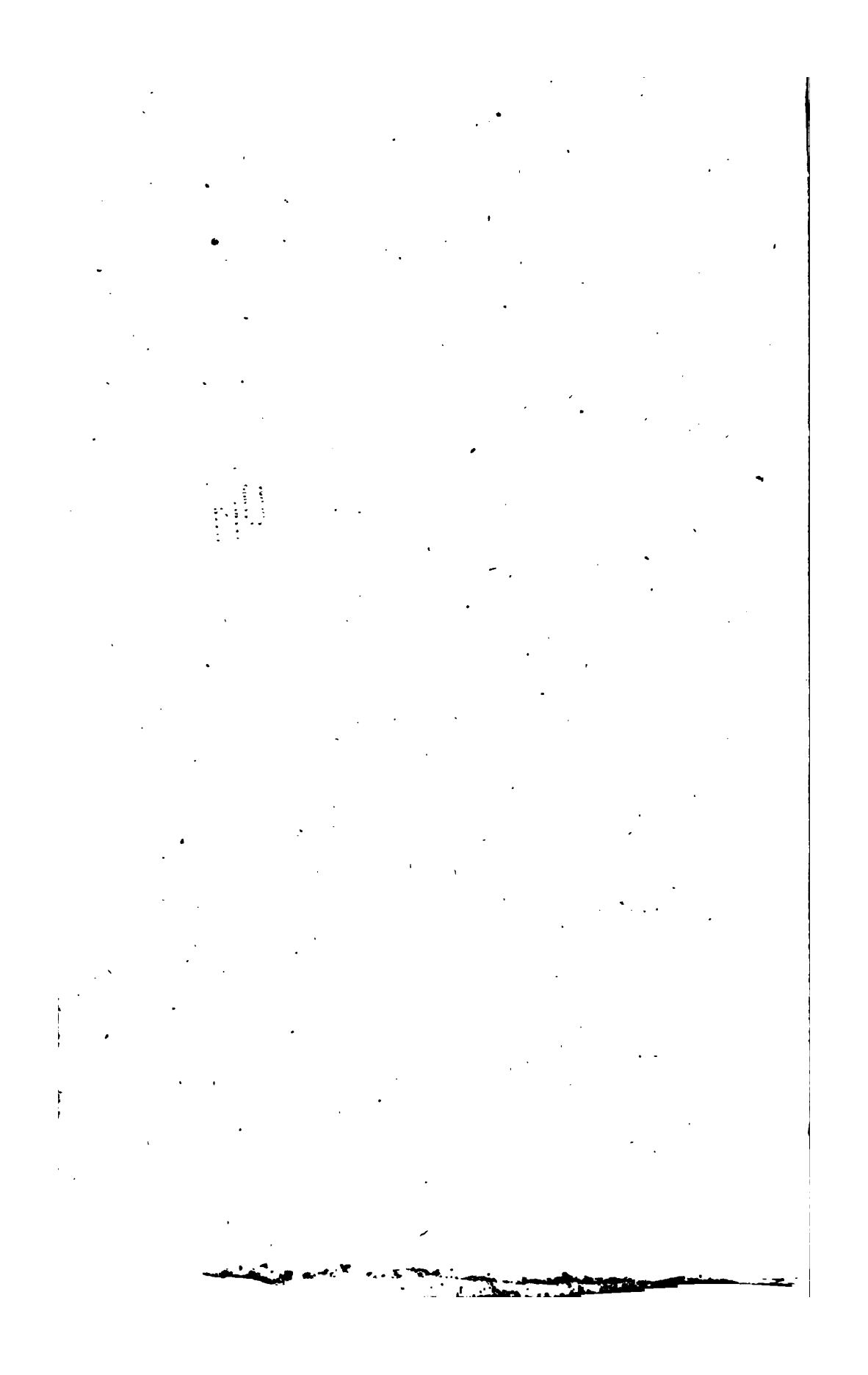


DEDICATION.

TO THE RIGHT HONOURABLE,

FRANCIS Lord NAPIER.

MY LORD,

IT may be unuſual, that a perſon who hath not had the honour of your Lordſhip's acquaintance, ſhould, in this public manner, crave your protection, in his attempt to promote the knowledge of that ſcience, whereby human life, which is liable to ſo many accidents, is yet, in ſome meaſure, reduced to rule. It flows, my Lord, from the reſpect which I entertain for the memory of your illuſtrious progenitor, John Napier, Baron of Merchiſton, in Scotland, whom I place upon

upon an equal footing with the moſt diſtinguiſhed characters of the ſeventeenth century, for his valuable invention of Logarithms, that I preſume to aſk your patronage in the publication of the following Treatiſe, and to profeſs myſelf, with all reſpect,

My LORD,

Your Lordſhip's moſt obedient,

And humble ſervant,

DAVID WILKIE.

PRE-

THE PREFACE.

SIMILAR to the ſatisfaction which the mind receives in the inveſtigation of geometrical propoſitions, is the pleaſure which we derive from the reſolution of queſtions in algebra, both on account of their ingenuity, certainty, and extenſive uſefulneſs. In the following Treatiſe, wherein ſo many algebraic theorems occur, it is propoſed to inveſtigate, from firſt principles, the rules adapted to one particular branch of ſcience, that of Intereſt, both ſimple and compound, with their application to Annuities of all deſcriptions; hoping thereby, not only to afford pleaſure to the ſtudious, in the inveſtigation of the theorems, but advantage alſo to men of buſineſs, in the application of theſe to the affairs of property.

The theorems, ſuch as $\frac{a}{r}$ being the value of a perpetuity, are as beautiful as any which are to be met with in the other branches of ſcience; and to render them the more intelligible, a demonſtration is ſubjoined where it is thought neceſſary. The tables are accurate, and ſeveral of them new, ſuch as Table IV. VIII. and XI. of annuities on lives; the firſt expreſſing the amount of L. 1 annuity upon a ſingle life; in the conſtruction

tion of which, the reaſon is diſcovered, why the value of a ſingle life is leſs, and its amount greater, than that correſponding to its expectation; and the two laſt ſhewing the probabilities of life adapted to Scotland. The conſtruction of the tables is clearly pointed out; and, by three equations annexed to each, all their uſes are diſcovered at firſt ſight. Several branches of education, connected with this ſcience, are treated of, ſuch as Arithmetical and Geometrical Progreſſions, the Extraction of the Square Root, Logarithms, the Arithmetic of Infinities, Fluxions, and the rule of Double Poſition.

In this Treatiſe there are many things new. The application of geometry, for amuſement, to matters of intereſt, moſt of the remarks annexed to each ſubject, the bearer's loſs in diſcounting bills, ſeveral of the theorems, ſuch as $P = \frac{PR - np}{nr}$, expreſſing the value of a ſingle life, a Tontine, and Inſurance from Fire, are all new. But what I chiefly value in this reſpect, are Prob. III. and XVIII. of Annuities on Lives; the former diſcovering the value, and the latter the expectation of the joint continuance, or of the longeſt liver of any number of lives, by methods which are both accurate, and ſave many pages of algebraic inveſtigation. As the queſtions in Annuities on Lives are various as the complections of men, I have,

in

n the courſe of twenty-two problems, delineated a great number of thoſe which are marked with the ſtrongeſt features. Particular attention hath been paid to the Widows Scheme in the Church of Scotland, wherein ſo many are intereſted; and the account given of its conſtruction and operation may enable Contributors to judge for themſelves, ſhould the affairs of the Fund ever come before them in a judicial capacity.

In the conſtruction of the theorems of Intereſt and Annuities certain, I acknowledge myſelf much indebted to the labours of that able algebraiſt Mr Nicolas Vilant, St Andrew's. Wherein I differ from others, as in the conſtruction of a widows ſcheme, in Prob. VIII, let the learned judge for themſelves; and wherein I am defective, I crave their indulgence. But I hope it will not be reckoned a defect, that I have not expreſſed the theorems in ſo many words, being perſuaded that an algebraic expreſſion is much more intelligible at firſt ſight, than it can be made by any language.

The ſcience of Annuities on Lives is yet in its infancy. Dr Halley was the firſt who conſtructed a table of the Probabilities of Life: Moivre, Dodſon, Webſter, and Price, theſe fathers of the ſcience, dropped only a few years ago. Thoſe theo-

rems of Mr De Moivre, which are built upon the principle, " that the decrements of life are in a " conſtant ratio," though beautiful in theory and eaſy in practice, are now exploded, becauſe they give the value of life too ſmall. In Scotland, we have no writer who treats methodically of Annuities; and Engliſh authors on theſe ſubjects are in very few hands: therefore, till a better appears, it is hoped that this Treatiſe, which has for its object a more perfect Syſtem of Intereſt and Annuities than has hitherto appeared, will not be deemed an unneceſſary performance.

CULTS,
October 18. 1792.

CON-

CONTENTS.

CHAPTER I.

Pag.

Arithmetical Progreſſion, with a Table and Examples, - - - 1

CHAPTER II.

Simple Intereſt, with Examples, 7

Rebate or Diſcount, - - 18
Four Tables adapted to Simple Intereſt, 21
Remarks on Simple Intereſt, - - 27

Supplement to CHAP. II.

1. Of Intereſt due upon Caſh-accounts, - 30
2. Of the Equation of Payment, - - 33
3. Of the Premium of Commiſſion, - 34

CHAPTER III.

Annuities computed at Simple Intereſt, 35

1. The amount of an Annuity in Arrears, - 36

Pag.
2. The preſent worth of an Annuity, - - 38
3. ——— ——— ——— in Reverſion, 40
4. ——— ——— of the Reverſion of an Annuity, ib.
Two Tables of Annuities at Simple Intereſt, - 41
Remarks upon Annuities at Simple Intereſt, - 45

CHAPTER IV.

Geometrical Progreſſion, with a Table and Examples, - - - 48

Of a Geometrical Series continued indefinitely, - 54

CHAPTER V.

Compound Intereſt, with Examples, 55

The method of Calculation by Logarithms, - 57
Three Tables, adapted to Compound Intereſt, - 59
Remarks upon Compound Intereſt, - 65

CHAPTER VI.

Annuities computed at Compound Intereſt, 67

1. The amount of an Annuity in Arrears, - ib.
The Rule of Double Poſition, - - 69
2. The Preſent worth of an Annuity, - - 73
3. ——— ——— of an Annuity in Reverſion, 77
The Renewing of Leaſes, with a Table, - 81
4. The preſent Worth of the Reverſion of an Annuity, 82
Three Tables of Annuities at Compound Intereſt, - 84
Annuities computed at Compound Intereſt when the Payments are made per advance, - - 94
Remarks upon Annuities at Compound Intereſt, - 96

Supplement

Pag.

Supplement to CHAP. VI.

Of a Sinking Fund to extinguiſh the National Debt, 97

CHAPTER VII.

The Value of a Perpetuity or Freehold Eſtate, - - - 101

Of a Reverſion in Perpetuity, - - 103
Remarks upon the Purchaſe of a Perpetuity, - 105
Three Additional Tables of Compound Intereſt, - 108

CHAPTER VIII.

Annuities on Lives, 110

Dr Halley's Table of the Probabilities of Life, 111
Six Articles derived from it, - - 112
Mr De Moivre's Hypotheſis of equal Decrements, 116

Twenty-two Problems derived moſtly from it.

PROB. I. To find the Value of a ſingle Life, - 117
PROB. II. ——— ——— of two joint Lives, 119
PROB. III. ——— ——— of the joint continuance, or of the Longeſt Liver of any number of Lives, 122
Ten Tables adapted to Annuities on Lives, - 127
PROB. IV. ——— ——— of the longeſt Liver of two Lives, - - - 144
PROB. V. ——— ——— of the longeſt of three Lives, - - - - 146
PROB. VI. ——— ——— for a given term of Years, on the Contingency of an aſſigned Life, with a Table, - - - - - 147

Supplement to PROB. I. II. III. IV. V. and VI.

To find, by a Geometrical Figure, the Value of Life, in four Articles, - - - 150

Art.

Pag.
Art. 1. To find the Value or amount of a Single Life, ib.
— 2. ——— or ——— of the joint continuance of any number of Lives, - - - 152
— 3. ——— or ——— of the longeſt Liver of ditto, 154
— 4. ——— or ——— for a given term of Years, on the Contingency of an Aſſigned Life, - 158

Of REVERSIONS.

Prob. VII. To find the value of a Reverſion in Perpetuity, after one or more aſſigned lives, - 162
Prob. VIII. To find the Value of the Reverſion of an Annuity during a given Life, after the Deceaſe of the preſent Poſſeſſor, - - - 165
Of a Widows Scheme, - - 167
Of a Marriage-Tax, - - 170
Prob. IX. ——— ——— during two joint Lives, after the Deceaſe of the preſent Poſſeſſor, - 172
Prob. X. ——— ——— during one Life, after two joint ones, - - 173
Prob. XI. ——— ——— during a Life of a given Age, after the longeſt of two Lives, - 174
Prob. XII. ——— ——— during the longeſt of two Lives after one, - - - 175
Prob. XIII. ——— ——— where a given Term of Years is concerned, - - 176
Prob. XIV. ——— ——— depending on Survivorſhip, and the Expectation of Life, - 180
Prob. XV. Of Inſurance on Lives, - 184
Prob. XVI. To find the Value of ſucceſſive Lives, 187
Prob. XVII. Of a Copyhold, and renewing of Leaſes, 189
Prob. XVIII. Of the Expectation of Life, - 192
The Arithmetic of Infinities, - 194
The Expectation of Life diſcovered by Fluxions, 199
Supplement to Prob. XVIII. - 200
Prob. XIX. Of the Probability of Survivorſhip, 206
Prob. XX. Of Half-yearly and Quarterly Payments of an Annuity, - - 212

Pag.
PROB. XXI. Of the Britiſh State-Lottery, - - 214
PROB. XXII. Of a Tontine, - 220

Supplement to CHAP. VIII.

Of Inſurance from Fire, 223

CHAP. IX.

Of the Widows Scheme in the Church of Scotland, contained in Twelve Articles, 225

Supplement to CHAP. IX.

A Diſſertation, reſpecting the Diſpoſal of the Surplus in the Fund of the Widows Scheme, when the Capital is raiſed to L. 100,000; addreſſed to the Truſtees for managing ſaid Fund, - - 254

The Signs which occur, without being explained in the body of the Work, are ſuch as theſe:

$=$ The Sign of Equality.

$+$ ——— Addition.

$-$ ——— Subtraction.

$\times$ ——— Multiplication.

$\frac{a}{b}$, or $a \div b$ — Diviſion; a divided by b.

$\overline{a+b}$, The Sum of the two Quantities a and b.

$\overline{a-b}$, The Difference of the two Quantities a and b.

$a : b :: c : d$, Ratio; a is to b as c is to d.

$\sqrt{a}$, The Square Root of a.

$\sqrt[n]{a}$, The n Root of a.

$\overline{a+b}^2$, The Square of $\overline{a+b}$.

R^n, The n Power of R.

$\frac{1}{R^n}$, 1 divided by the n power of R.

L. The Logarithm of a Quantity.

CHAPTER I.

Of ARITHMETICAL PROGRESSION.

1. WHEN a ſeries of quantities, exceeding two in number, increaſes or decreaſes by the addition or ſubtraction of one common difference, the terms of the ſeries are ſaid to be in continued arithmetical progreſſion. Thus,

1, 2, 3, 4, 5, 6, 7, 8, 9, 10. an increaſing ſeries.
10, 9, 8, 7, 6, 5, 4, 3, 2, 1. a decreaſing ſeries.

$a,\ a+d,\ a+2d,\ a+3d,\ a+4d \ldots a+\overline{n-1} \times d = v.$
$v,\ v-d,\ v-2d,\ v-3d,\ v-4d \ldots v-\overline{n-1} \times d = a.$

$a+v,\ a+v,\ a+v,\ a+v,\ a+v \ldots \qquad a+v = \left\{\begin{matrix}\text{Sum of}\\ \text{both.}\end{matrix}\right.$

2. If four quantities are thus proportional, the ſum of the extremes will be equal to the ſum of the means; Thus,

$a,\ a+d,\ a+2d,\ a+3d$; then $2\ a+3d = \left\{\begin{matrix}\text{Sum of the extremes.}\\ \text{Sum of the means.}\end{matrix}\right.$

3. If three quantities are thus proportional, the

ſum of the extremes is double of the mean; Thus,

a, $a+d$, $a+2d$; then $2\ a+2d = \begin{cases} \text{Sum of the extremes.} \\ \text{Double of the mean.} \end{cases}$

4. Whatever be the number of terms in an arithmetical progreſſion, the ſum of the two extremes will be equal to the ſum of any two terms equidiſtant from theſe extremes. Thus, $1+10 = 5+6 = 3+8$.

5. In a ſeries of terms in arithmetical progreſſion, let

a = the firſt term.
v = the laſt term.
n = the number of terms.
d = the common difference.
s = the ſum of the ſeries; then as d is found in all the terms except the firſt, its co-efficient in the laſt term will be $n-1$, hence $a+\overline{n-1} \times d = v$, $v-\overline{n-1} \times d = a$, and $\overline{n-1} \times d = a \sim v$.

6. In any number of terms in arithmetical progreſſion, the firſt and laſt terms, multiplied by the number of terms, are equal to double the ſum of the whole ſeries; Thus in the two ſeries above (1) which are perfectly equivalent, it is manifeſt that the two taken together, that is the double of any one of them, will be equal to the ſum of a ſeries of equals $\overline{a+v}$, $\overline{a+v}$ &c. taken to n terms $= \overline{a+v} \times n = 2s$.

7. Therefore, in a continued arithmetical progreſſion, if any three of the five quantities, a, v, d, n, and s, are ſuppoſed to be given, the remaining two may be found, as in the following table:

Given.

Given.		
$a, d, n,$	$s = n \times \overline{a + d \times \frac{n-1}{2}}, \ldots$	Equation 1.
$a, d, v,$	$s = \frac{v+a}{2} \times \overline{\frac{v-a}{2} + 1}, \ldots$	2.
$a, n, v,$	$s = \overline{a+v} \times \frac{n}{2}, \ldots$	3.
$d, n, v,$	$s = n \times \overline{v - d \times \frac{n-1}{2}}, \ldots$	4.
$a, d, n,$	$v = a + \overline{n-1} \times d, \ldots$	5.
$a, d, s,$	$v = \sqrt{a \times \overline{a-d} + d \times \overline{2s + \frac{d}{4}}} - \frac{1}{2}d, \ldots$	6.
$a, n, s,$	$v = \frac{2s}{n} - a, \ldots$	7.
$d, n, s,$	$v = \frac{s}{n} + \frac{1}{2}d \times \overline{n-1}, \ldots$	8.
$d, v, n,$	$a = v - \overline{n-1} \times d, \ldots$	9.
$d, n, s,$	$a = \frac{s}{n} - \frac{1}{2}d \times \overline{n-1}, \ldots$	10.
$n, v, s,$	$a = \frac{2s}{n} - v, \ldots$	11.
$d, v, s,$	$a = \frac{1}{2}d \pm \sqrt{v \times \overline{v+d} - d \times \overline{2s + \frac{d}{4}}}, \ldots$	12.

ught s in Equation 1st.

:. 6. $2s = n \times \overline{a+v}$

$s = n \times \frac{a+v}{2}$

:. 5. $v = a + d \times \overline{n-1}$

$s = n \times \frac{2a + d \times \overline{n-1}}{2}$

:nce $s = n \times \overline{a + d \times \frac{n-1}{2}}$

Sought v in Equation 6.

Sec. 5. $\overline{a+v} \times n = 2s$

$n = \frac{v-a}{d} + 1$

$\frac{v^2 - a^2}{d} + v + a = 2s$

$v^2 - a^2 + dv + da = 2ds$

$v^2 + dv = a \times \overline{a-d} + 2ds$

$v^2 + dv + \frac{1}{4}d^2 = a \times \overline{a-d} + d \times \overline{2s + \frac{d}{4}}$

$v + \frac{1}{2}d = \sqrt{a \times \overline{a-d} + d \times \overline{\ldots}}$

Given.		
a, d, v,	$n = \frac{v-a}{d} + 1, \ldots$	Equation 13.
a, d, s,	$n = \sqrt{\frac{2s-a}{d} + \frac{a^2}{d^2} + \frac{1}{4}} - \frac{a}{d} + \frac{1}{2}, \ldots$	14.
a, v, s,	$n = \frac{2s}{v+a}, \ldots$	15.
d, v, s,	$n = \frac{v}{d} - \frac{1}{2} + \sqrt{\frac{v^2}{d^2} + \frac{v}{d} + \frac{1}{4} - \frac{2s}{d}} + \frac{1}{2}, \ldots$	16.
a, v, s,	$d = \frac{\overline{v+a} \times \overline{v-a}}{2s - v + a}, \ldots$	17.
a, n, s,	$d = \frac{1}{n-1} \times \frac{2s-2an}{n}, \ldots$	18.
a, n, v,	$d = \frac{v-a}{n-1}, \ldots$	19.
n, v, s,	$d = \frac{2v - 2s\frac{1}{n}}{n-1}, \ldots$	20.

Sought n in Equation 14th.

Sec. 5. and 6.

$$\overline{2a+dn-d} \times n = 2s$$

$$2an + dn^2 - dn = 2s$$

$$n^2 + \frac{2a}{d}n - n = \frac{2s}{d}$$

$$n + \frac{a}{d} - \frac{1}{2} = \sqrt{\frac{2s-a}{d} + \frac{a^2}{d^2} + \frac{1}{4}}$$

Sought n in 16th.

Sec. 5. and 6.

$$2vn - dn - dn^2 = 2s$$

$$\frac{2v}{d}n + n - n^2 = \frac{2s}{d}$$

$$\left.\begin{array}{l} \overline{\qquad\qquad}^2 - \frac{2v}{d} \\ \overline{\frac{v}{d} + \frac{1}{2} - 1} \end{array}\right\} n + n^2 = \overline{\frac{v}{d} + \frac{1}{2}}^2 - \frac{2s}{d}$$

$$\frac{v}{d} + \frac{1}{2} - n = \&c.$$

Sought d in 17th.

Sec. 6.

$$\overline{v+a} \times \frac{\overline{v-a}}{d} + 1 = 2s$$

$$\frac{\overline{v+a} \times \overline{v-a}}{d} + v + a = 2s$$

$$\overline{v+a} \times \overline{v-a} = 2ds - d \times \overline{v+a}$$

Sought d in 18th.

$$\overline{2a + \overline{n-1} \times d} \times n = 2s$$

$$2an + \overline{n-1} \times dn = 2s$$

$$\overline{n-1} \times d = \frac{2s-2an}{n}$$

Sought d in 20th.

$$\overline{2v - \overline{n-1} \times d} \times n = 2s$$

$$2vn - n - 1 \times dn = 2s$$

$$\overline{n-1} \times d = 2v - 2s\frac{1}{n}$$

In

In the above table, the expreſſions belong to an increaſing progreſſion, and may be applied alſo to a decreaſing one, by making the quantities a and v change places every where.

EXAMPLES.

1*ſt*, Sought, the ſum of the firſt hundred numbers in their natural order. Here, $a=1$, $n=100$, $v=100$; by equation 3d, $\overline{a+v} \times \frac{n}{2} = 101 \times 50 = 5050 =$ ſum ſought.

2*d*, Suppoſe a baſket and 500 apples were placed in a ſtraight line, a yard diſtant from each other, it is required how far one muſt go before he brings them, one by one, into the baſket? Here, as the diſtance muſt be gone over twice in bringing each apple, $a=2$, $d=2$, $n=500$; by equation 1ſt, $n \times \overline{a+d \times \frac{n-1}{2}} = 500 \times 501 = 250500$ yards $=$ 142 miles.

3*d*, A man had 12 children; the youngeſt was three years old, and the common difference of their ages was four years; What was the age of the eldeſt? Here $a=3$, $d=4$, $n=12$. By equation 5th, $a + \overline{n-1} \times d = 3+44 = 47$ years.

8. When a number of terms in arithmetical progreſſion begins with a cypher, (0), which is the moſt regular ſeries, in this caſe, the ſum of the ſeries is equal to the laſt term multiplied by half the number of terms. For $a=0$, and, equation

tion 3d, $s = v \times \frac{n}{2}$. Thus 0, 1, 2, 3, ... 31 $= 31 \times 16 = 496$.

9. When $a = 2$, and $d = 2$ alſo, in this caſe, in equation 1ſt, $s = n^2 + n = a$ pronic number, which is produced by the addition of even numbers in an arithmetical progreſſion, beginning at 2; and the pronic root $n = \frac{\sqrt{4s+1} - 1}{2}$.

10. When $a = 1$ and $d = 2$, then $s = n^2$. So that all ſquare numbers may be conſidered as ariſing from the addition of odd numbers in an arithmetical progreſſion. Thus.

Arithmetical Progreſſion, 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, &c.

Square Numbers, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, &c.

Hence may be determined how many odd numbers muſt be added to produce a given power of that number. Let the given number be n, the exponent of its power m, and the firſt term of the ſeries a; then ſince $d = 2$, and the number of terms n, hence $s = n^m = n \times \overline{a + n - 1}$ and $a = n^{m-1} - \overline{n-1}$. And thus it appears, how powers of all dimenſions may be obtained by ſumming up of odd numbers. Thus, ſuppoſing $m = 3$, and $n = 3, 4, 5, 6$, &c.

$3^3 = 7 + 9 + 11$
$4^3 = 13 + 15 + 17 + 19.$
$5^3 = 21 + 23 + 25 + 27 + 29.$
$6^3 = 31 + 33 + 35 + 37 + 39 + 41.$

CHAP.

CHAP. II.

Of SIMPLE INTEREST.

11. WHEN a ſum of money is lent upon intereſt, and that intereſt is either kept in the borrower's hands, or is paid regularly, without becoming a part of the principal, it is then ſaid to bear ſimple intereſt.

The intereſt of money was formerly very high; but by an act of parliament, made in 12th Queen Anne, it is now reduced to, or it muſt not exceed, 5 per cent. per annum. As the intereſt of L. 1 for one year is ſo neceſſary in computations both of ſimple and compound intereſt, it may be found, for any rate of intereſt per cent, by the following proportions:

	3	.03	$\frac{1}{33.3}$		3
	3.5	.035	$\frac{1}{28.57}$		$3\frac{1}{2}$
	4	.04	$\frac{1}{25}$		4
As 100:	4.5, ſo is 1:	.045 =	$\frac{1}{22.2}$ =	Intereſt of L. 1 for one year at	$4\frac{1}{2}$ per cent.
	5	.05	$\frac{1}{20}$		5
	6	.06	$\frac{1}{16.6}$		6
	8	.08	$\frac{1}{12.5}$		8
	10	.1	$\frac{1}{10}$		10

12.

12. In ſimple intereſt, let

p, repreſent any principal lent out upon intereſt.

r, —— —— the intereſt of L. 1 for one year.

t, —— —— the time.

d, —— —— any number of days.

i, —— —— the intereſt, and

s, —— —— the amount of principal and intereſt.

Then, r, $2r$, $3r$, $4r$, ... tr, will be the intereſt of L. 1 for 1, 2, 3, 4, ... t times; but $1 : tr :: p : trp = i$, the intereſt of p for t times at the rate r; and $\frac{drp}{365}$ = intereſt of p for d days at the rate r. Hence the following

TABLE.

Given.		
$t, r, p,$	$i = trp = \frac{tp}{20}$ or $\frac{tp}{25}$ &c.	Equation 1.
$i, t, r,$	$p = \frac{i}{tr}$	2.
$i, r, p,$	$t = \frac{i}{rp}$	3.
$i, t, p,$	$r = \frac{i}{tp}$	4.
$d, r, p,$	$i = \frac{drp}{365}$	5.

EX-

EXAMPLES.

1*ſt*, Sought, the intereſt of L. 356 lent for 12 years, at the rate of $4\frac{1}{2}$ per cent. per annum. Here $trp = 356 \times .54 = 192.24$ = L. 192, 4s. $9\frac{1}{2}$d. = intereſt.

2*d*, Sought, the principal whoſe intereſt will amount to L. 145 in the ſpace of 9 years, at the rate of 5 per cent. Here $\frac{i}{tr} = \frac{145}{.45}$ = L. 322, 4s. $5\frac{1}{2}$d. = principal.

3*d*, Sought, the time in which L. 567, 10s. will produce L. 306, 9s. of intereſt, at the rate of 6 per cent. Here $\frac{i}{rp} = \frac{306.45}{34.05} = 9$, the years required.

4*th*, Sought, the intereſt of I.. 378, 14s. for the ſpace of 127 days, at 5 per cent. Here $\frac{drp}{365}$ $= \frac{2404.7}{365}$ = L. 6 : 11 : 9 = intereſt required.

13. Again, the intereſt of any principal p, being manifeſtly in the compound ratio of p the principal, t the time, and r the rate; it will be, as above, trp; to which adding p, we ſhall have s, the amount of principal and intereſt $= p + trp = \overline{1+tr} \times p$. Hence the following table.

Given.		
$p, t, r,$	$s = p + trp = \overline{1+tr} \times p \ldots$	Equation 1.
$s, t, r,$	$p = \frac{s}{1+tr} \ldots$	2.
$s, r, p,$	$t = \frac{s-p}{rp} \ldots$	3.
$s, p, t,$	$r = \frac{s-p}{tp} \ldots$	4.

EXAMPLES.

1*ſt*, Sought, the amount of L. 156, lent for 12 years, at the rate of $4\frac{1}{2}$ per cent. Here $\overline{1+tr} \times p = 1.54 \times 156 = 240.24 =$ L. 240 : 4 : $9\frac{1}{2}$ d. $= s$.

2*d*, Sought, the principal which will amount to L. 873, 19 s. in 9 years, at 6 per cent. Here $\frac{s}{1+tr} = \frac{873.95}{1.54} =$ L. 567, 10 s. the principal ſought.

3*d*, Sought, the time in which L. 600, 14 s. will amount to L. 871.015, at the rate of $4\frac{1}{2}$ per cent. Here $\frac{s-p}{rp} = \frac{270.315}{27.0315} = 10$ years.

14. If the principal or intereſt conſiſts of pounds, ſhillings, and pence, theſe parts of a pound may be thrown into a decimal fraction, and

and the intereſt or amount will be found as above. There is an eaſy method of converting the decimals of a pound into ſhillings and pence, viz. double the firſt decimal on the left hand, calling it ſo many ſhillings, to which add 1, if the 2d decimal exceeds 5; prefix the remainder above 5, in the 2d place of decimals, to the decimal in the 3d place, calling them ſo many farthings, and ſubtract 1 at or above 25, and 2 at or above 47, and *vice verſa*. Thus, L. .7856 = 15 s. 8½ d. and 11 s, 6½ d. = L. .578.

15. If the time *t* conſiſts of years, months, and days, or of days only, their number may be collected from the following table, ſhewing the number of days from the 1ſt, 10th, 20th, &c. of one month, to the 1ſt, 10th, 20th, &c. of any other month; where the months on the left hand column are the former dates, and thoſe at the top are the latter. Thus, from 1ſt of July to 1ſt of April following, are 274 days; and from 20th of March to 10th of Auguſt, are 153—10 = 143 days.

TABLE.

To 1st, &c. of.

From 1st, &c. of.	*M.*	*Jan.*	*Feb.*	*Mar*	*April*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
	Jan.	365	31	59	90	120	151	181	212	243	273	304	334
	Feb.	334	365	28	59	89	120	150	181	212	242	273	303
	March.	306	337	365	31	61	92	122	152	184	214	245	275
	April.	275	306	334	365	30	61	91	122	153	183	214	244
	May.	245	276	304	335	365	31	61	92	123	153	184	214
	June.	214	245	275	304	334	365	30	61	92	122	153	183
	July.	184	215	243	274	304	335	365	31	62	92	123	153
	August.	153	184	212	243	273	304	334	365	31	61	92	122
	Sept.	122	153	181	212	242	273	303	334	365	30	61	91
	Oct.	92	123	151	182	212	243	273	304	335	365	31	61
	Nov.	61	92	120	151	181	212	242	273	304	334	365	30
	Dec.	31	62	90	121	151	182	212	243	274	304	335	365
	Days.	31	28	31	30	31	30	31	31	30	31	30	31

16. Theſe days may be thrown into the decimals of a year by the following Table.

Days.	Decimals	Days.	Decimals.	Days.	Decimals.
1	.0027397	8	.0219178	60	.1643835
2	.0054794	9	.0246575	70	.1917808
3	.0082192	10	.0273972	80	.2191780
4	.0109589	20	.0547945	90	.2465753
5	.0136986	30	.0821918	100	.2739726
6	.0164383	40	.1095890	200	.5479452
7	.0191780	50	.1369863	300	.8219178

Thus it is required to expreſs 143 days in the decimal fraction of a year.

100 = .2739726
40 = .1095890
3 = .0082192

143 = .3917808 of a year.

Ex. Required the intereſt of L. 356, 12 s. for 3 years and 143 days, at five per cent. per annum. Here

$trp = 3.41178 \times .05 \times 356.6 = $ L. 60 : 16 : $7\frac{1}{2} = $ Intereſt.

17. The intereſt of any ſum of money, for any number of years, at a given rate, may be collected from the following table, calculated for the nine digits, at ſeveral rates of ſimple intereſt.

TABLE.

	trp for years.					
tp	3 pr cent	3½ pr cent	4 pr cent	4½ pr cent	5 pr cent	6 pr cent
1	.03	.035	.04	.045	.05	.06
2	.06	.07	.08	.09	.10	.12
3	.09	.105	.12	.135	.15	.18
4	.12	.14	.16	.18	.20	.24
5	.15	.175	.20	.225	.25	.30
6	.18	.21	.24	.27	.30	.36
7	.21	.245	.28	.315	.35	.42
8	.24	.28	.32	.36	.40	.48
9	.27	.315	.36	.405	.45	.54

In this table, the decimals, for inſtance, correſponding to 9 expreſs the intereſt of L. 9 for 1 year, or of L. 1 for 9 years, or of *p* for *t* years, whatever

whatever theſe be, provided their product $tp=9$, at a given rate. Of conſequence the intereſt-colums are equal to trp, while the marginal one correſponds to tp. And it is evident that the tabular numbers will not only anſwer for units, but alſo for tens, hundreds, or thouſands, by removing the decimal point one, two, or three places to the right hand. Hence, in uſing this table, multiply the principal by the number of years in queſtion, and take out the decimals correſponding to their product; theſe being properly pointed and added together will expreſs the intereſt required.

Ex. Required the intereſt of L. 348, 12 s. for 12 years, at four per cent. per annum. Here $348.6\times12=4183.2$.

By the Table.

4000	=	160.0
100	=	4.0
80	=	3.2
3	=	.12
.2	=	.008

L. 167 : 6 : $6\frac{3}{4}$ = 167.328 = Intereſt.

If the given rate be not in the table, the intereſt may be found by reduction.

18. The intereſt of any ſum of money, for any number of days or months, at a given rate, may be collected from the following table, calculated for the 9 digits, at ſeveral rates of ſimple intereſt.

TABLE.

TABLE.

	$\frac{drp}{365}$ for Days.			$\frac{mrp}{12}$ for Months.		
dp	4 per cent.	4½ per cent.	5 per cent.	4 pr cent	4½ p. cent	5 pr cent.
1	.00010959	.00012328	.00136986	.0033	.00375	.0041β
2	.00021918	.00024657	.00273972	.006β	.0075	.0083
3	.00032876	.00036986	.00410958	.0100	.01125	.01250
4	.00043835	.00049315	.00054794	.013	.01500	.0166β
5	.00054794	.00061643	.00068493	.016β	.01875	.0208
6	.00065753	.00073972	.00082191 8	.0200	.02250	.02500
7	.00076712	.00086301	.00095890	.023	.02625	.0291β
8	.00087671	.00098629	.0010959	.026β	.03000	.0333
9	.00098630	.00110958	.00123287	.0300	.03375	.0375

In this table alſo, the decimals, for inſtance, correſponding to 6, expreſs the intereſt of L. 6 for 1 day, at a given rate, or of L. 1 for 6 days, or of p for d days, whatever theſe be, provided their product $dp=6$. Of conſequence the intereſt-columns are equal to $\frac{drp}{365}$, while the marginal one correſponds to dp. The ſame of months. In computing intereſt by days, which is more uſual than by months, the year is always ſuppoſed to conſiſt of 365 days; the 29th of February, in leap years, not being taken into the account. Alſo the day computed from is not reckoned, but the day computed to is: and in diſcharging bills, three days of grace are allowed the acceptor to make payment.

In uſing the above table, multiply the principal by the number of days or months in queſtion, and take out the decimals correſponding to their product;

product; these being properly pointed and added together will express the interest required.

Ex. Required the interest of L. 564 for 238 days, at $4\frac{1}{2}$ per cent. per annum. Here $564 \times 238 = 134232$.

By the Table.

100000 =	12.3287
30000 =	3.6986
4000 =	.49315
200 =	.02465
30 =	.00369
2 =	.00024
L. 16, 11 s. =	16.54903 = Interest sought.

If the given rate be not in the table, the interest may be found by reduction; or by the following table.

19. The interest of any sum of money, for any number of days, and at any given rate, may be collected from the following table.

TABLE.

drp	$\frac{drp}{365}$ for Days.
1	.002739726
2	.005479452
3	.008219178
4	.010958904
5	.013698630
6	.016438356
7	.019178082
8	.021917808
9	.024657534

In this table, the digits in the first column are the products drp; and the decimals are the quotients of these products divided by 365, and, of consequence, are the interest of p for d days, and at the rate r, whatever these factors be. Therefore to find the interest of any sum of money by the table, multiply

multiply the principal, the number of days, and the rate of L. 1 per annum, into one ſum, and from the table take out the intereſt correſponding to their product.

Ex. Required the intereſt of L. 342, 15 s. for 260 days, at 5 per cent. Here $342.75 \times 260 \times .05 = 4455.75$.

By the Table.

4000	=	10.9589
400	=	1.09589
50	=	.13698
5	=	.01369
.7	=	.00192
.05	=	.000137

L. 12 : 4 : 2 = 12.207517 = Intereſt.

20. Or the intereſt of any ſum of money, for any number of days, at five per cent. may be diſcovered by the following proportion:

$100 \times 365 : 5 :: p \times d : \frac{p \times d}{7300}$. Hence the following equations,

1ſt, $\frac{p \times d}{7300}$ = Intereſt of p for d days,
2d, $\frac{p}{73}$ = Intereſt of p for 100 days,
3d, $\frac{d}{73}$ = Intereſt of L. 100 for d days,
} at 5 per cent.

4th, When $\frac{d}{73} = 1, 2, 3$, &c. then $\frac{p}{100}$ = intereſt of p for $73 \times 1, 2, 3$, &c. days.

Ex. 1ſt. Suppoſe p = L. 564, and d = 238 days; then $\frac{p \times d}{7300} = \frac{134232}{7300} = 18.388$ = L. 18 : 7 : 9, the intereſt.

Ex. 2d, Suppoſe p=L. 342.75, and d=260 days; then,

$\frac{p}{73}=\frac{342.75}{73}=4.6952$

2

9.3904=Int. for 200 days.

$\frac{6}{10}=2.81712$

12.20752=Intereſt required.

Ex. 3d, Suppoſe p=L. 448, and d=236 days; then,

$\frac{d}{73}=\frac{236}{73}=3.23287$=Intereſt of L. 100.

4

12.93148

$\frac{4}{10}=1.29314$

$\frac{1}{5}=.25863$

14.48325 = Intereſt of L. 448.

Of Rebate or Diſcount.

21. To find p, the preſent value of any ſum s, due at any time t, and at any rate of intereſt r. Here (13, Eq. 2.) $\frac{s}{1+tr}=p$ the preſent value of s, and $s-p$=diſcount.

Ex. What ready money will pay a bill of L. 682, 18 s. due 3 years and 9 months hence, diſcounting intereſt at the rate of 5 per cent. Here $\frac{s}{1+tr}=\frac{682.9}{1.1875}=575.07=p$, and 682.9—575.07 = L. 107 : 16 : $7\frac{1}{4}$ the diſcount. If the time

time be days, they may be thrown into the decimals of a year by a former table (16.) and then you may proceed as above.

Ex. What is the preſent value of a bill of L. 285, 10 s. due 95 days hence, diſcounting intereſt at 5 per cent.

$95 = .26 = t$ $\quad$ $\frac{285.5}{1.013+} = 281.841 = p.$
$.05 = r$

$1.013 = 1 + tr$, $285.5 - 281.841 = 3.66 =$ diſcount.

Or thus, ſeeing $\frac{d}{73}$ = intereſt of L 100 for d days at 5 per cent. (20); hence, in the laſt example,

$\frac{2}{7}\frac{5}{3} = 1.3$ = Intereſt of L. 100 for 95 days;
then $101.3 : 1.3 :: 285.5 : 3.66$ = diſcount.

3.66

281.84 = preſent value.

But as the diſcounting bills in this manner is no better than lending money upon intereſt, bankers, and others who keep money for the purpoſe of diſcounting bills, conſider the ſum to be paid in the bill as a principal, find the intereſt thereof for the time to run, adding thereto three days of grace, and deducting the intereſt ſo found from the content of the bill, they pay the balance to him for whom the diſcount is made: ſome, likewiſe, for their trouble, charge $\frac{1}{4}$ or $\frac{1}{2}$ per cent. commiſſion, which, before deduction, is added to the intereſt.

Ex. A bill is preſented the 5th of May for diſcount, of L. 350, 16 s. payable 27th of July; how much money does the bearer receive, after

deduction of intereſt and $\frac{1}{4}$ per cent. commiſſion? Here $p=350.8$, $d=83+3$; hence

$\frac{350.8\times86}{7300}=4.133$=Intereſt.
.877=Commiſſion.

5.010=Sum to be deducted.
Hence L. 345.790 paid the bearer of the bill.

But the true diſcount being preciſely equal to the intereſt of the preſent value of the bill for the time to run, as calculated in a former example, it is evident that the bearer is a conſiderable loſer by this tranſaction; and it will be found, that, calling the intereſt of the bill its diſcount, the bearer thereof loſes exactly, beſides commiſſion, the intereſt of its true diſcount for the time to run. For calling d the true diſcount found by 13. Equation 2. then ſeeing $s=d+p$,

I—(intereſt of s=) intereſt of $d+p$, for the time of diſcount; but d=intereſt of p for the ſaid time: hence $i-d$, (the bearer's loſs when i is the diſcount)=Intereſt of d for the time of diſcount.

Ex. A bill of L. 210 payable a twelvemonth hence is preſented to be diſcounted; required the bearer's loſs in this tranſaction:

L. 10 10 the intereſt of the bill.
10 00 the true diſcount.

10 s.=the bearers loſs=intereſt of true diſcount=$\frac{1}{4}$ per cent. per annum, upon the preſent worth of the bill, for the time of diſcount. When the rate of intereſt is not mentioned, 5 per cent. the legal intereſt, is always underſtood.

22. The

22. The ſeveral particulars reſpecting ſimple intereſt will more readily be found by the help of the four following tables, wherein are calculated the amount and preſent value of L. 1 for days or years, at ſeveral rates of intereſt.

TABLE I. Shewing the Amount of L. 1 for Days.

D	4 per cent.	4½ per cent.	5 per cent.	D.	4 pr cent.	4½ pr cent.	5 pr cent.
1	1.00010958	1.00012328	1.00013698	27	1.002959	1.003328	1.003698
2	1.00021917	1.00024657	1.0002739	28	1.003068	1.003452	1.003835
3	1.00032876	1.00036986	1.0004109	29	1.003178	1.003575	1 003972
4	1.00043834	1.00049314	1.0005479	30	1.003287	1.003698	1.004109
5	1.00054794	1.00061643	1.0006849	31	1.003397	1.003822	1.004246
6	1.00065752	1 00073972	1.0008219	32	1.003507	1.003945	1.004383
7	1.00076710	1.00086300	1.0009589	33	1.003616	1.004068	1.004520
8	1.00087668	1.00098628	1.001958	34	1.003726	1.0041917	1.004657
9	1.00098628	1.0011095	1.0012328	35	1.003835	1.004315	1.004794
10	1.0010958	1.0012328	1.0013698	36	1.003945	1.004438	1.004931
11	1.0012059	1.0013561	1.0015068	40	1.004383	1.004931	1.005479
12	1.0013150	1.0014794	1.0016438	50	1.005479	1.006164	1.006849
13	1 0014246	1.0016027	1.0017808	60	1.006575	1 007397	1.008219
14	1.0015342	1 0017260	1.0019178	70	1 007611	1.008630	1.009589
15	1.0016438	1.0018493	1.0020548	80	1.008767	1.009862	1.010959
16	1.0017534	1.001972	1.0021917	90	1.009863	1.011095	1.012328
17	1.0018629	1.0020958	1.0023287	100	1.010958	1.012328	1.013698
18	1.0019726	1.002219	1.0024657	200	1.021917	1.02465	1.027397
19	1.0020821	1.002242	1.0026027	300	1.03287	1.03697	1.041095
20	1.00219178	1.002465	1.0027397	310	1.03397	1 03822	1.042465
21	1.0023013	1.002589	1.0028767	320	1.03506	1.03945	1.043835
22	1.0024109	1.002712	1.0030137	330	1.03616	1.04068	1.045205
23	1.0025205	1.002835	1.0031506	340	1.03726	1.041917	1.046575
24	1.0026301	1.002958	1.0032876	350	1.03835	1.04315	1.047945
25	1.0027397	1.003082	1.0034246	360	1 03945	1 04438	1.049315
26	1.0028493	1.003205	1.0035616	365	1.04	1.045	1.05

TABLE

TABLE II. Shewing the Amount of L. 1 for Years.

Yrs	4 pr cent.	4½ p. cen.	5 pr cent.	6 pr cent.	Yrs	4 pr cent.	4½ p. cen.	5 pr cent.	6 pr cent.
1	1.04	1.045	1.05	1.06	31	2.24	2.395	2.55	2.86
2	1.08	1.09	1.10	1.12	32	2.28	2.44	2.60	2.92
3	1.12	1.135	1.15	1.18	33	2.32	2.485	2.65	2.98
4	1.16	1.18	1.20	1.24	34	2.36	2.53	2.70	3.04
5	1.20	1.225	1.25	1.30	35	2.40	2.575	2.75	3.10
6	1.24	1.27	1.30	1.36	36	2.44	2.62	2.80	3.16
7	1.28	1.315	1.35	1.42	37	2.48	2.665	2.85	3.22
8	1.32	1.36	1.40	1.48	38	2.52	2.71	2.90	3.28
9	1.36	1.405	1.45	1.54	39	2.56	2.755	2.95	3.34
10	1.40	1.45	1.50	1.60	40	2.60	2.80	3.00	3.40
11	1.44	1.495	1.55	1.66	41	2.64	2.845	3.05	3.46
12	1.48	1.54	1.60	1.72	42	2.68	2.89	3.10	3.52
13	1.52	1.585	1.65	1.78	43	2.72	2.935	3.15	3.58
14	1.56	1.63	1.70	1.84	44	2.76	2.98	3.20	3.64
15	1.60	1.675	1.75	1.90	45	2.80	3.025	3.25	3.70
16	1.64	1.72	1.80	1.96	46	2.84	3.07	3.30	3.76
17	1.68	1.765	1.85	2.02	47	2.88	3.115	3.35	3.82
18	1.72	1.81	1.90	2.08	48	2.92	3.16	3.40	3.88
19	1.76	1.855	1.95	2.14	49	2.96	3.205	3.45	3.94
20	1.80	1.90	2.00	2.20	50	3.00	3.25	3.50	4.00
21	1.84	1.945	2.05	2.26	51	3.04	3.295	3.55	4.06
22	1.88	1.99	2.10	2.32	52	3.08	3.34	3.60	4.12
23	1.92	2.035	2.15	2.38	53	3.12	3.385	3.65	4.18
24	1.96	2.08	2.20	2.44	54	3.16	3.43	3.70	4.24
25	2.00	2.125	2.25	2.50	55	3.20	3.475	3.75	4.30
26	2.04	2.17	2.30	2.56	56	3.24	3.52	3.80	4.36
27	2.08	2.215	2.35	2.62	57	3.28	3.565	3.85	4.42
28	2.12	2.26	2.40	2.68	58	3.32	3.61	3.90	4.48
29	2.16	2.305	2.45	2.74	59	3.36	3.655	3.95	4.54
30	2.20	2.35	2.50	2.80	60	3.40	3.70	4.00	4.60

Tables I. and II. ſhew the amount of L. 1 for days or years, and are conſtructed by Equation 1. 13. whereby n the tabular number $= 1 + tr$. Or thus, Table I. is conſtructed by taking 364 mean arithmetical proportionals between 0 and the rate of L. 1 for one year, or by dividing this rate by 365, the quotient gives the intereſt of L. 1 for one day; and this intereſt added to itſelf continually gives the intereſt for any number of days, to which prefixing 1, you have the amount of L. 1 for theſe days.

days. If a given number of days are not in the table, divide them into ſuch numbers as are to be found there, and to the ſum of the decimals correſponding to theſe numbers prefix 1, the whole will expreſs the amount of L 1. for the given number of days at a given rate. Or remove the decimal point, where it can be done, one figure to the right hand, and you have the amount for 10 times the tabular number; thus 240=1.032876. at five per cent.——In like manner Table II. is conſtructed by adding the intereſt of L. 1 continually to itſelf for any number of years, to which adding 1, you have the amount of L. 1 for theſe years. If the given time conſiſts of years and days, to the tabular number correſponding to the years in Table II. add the decimals or intereſt anſwering to the days in Table I. their ſum will be the amount of L. 1 for the given time. From the nature of theſe two tables, $1 : n :: p : np=s$ the amount of p, for the time and at the rate correſponding to n. Hence the following table:

TABLE.

Given.	
$p, t, r,$	$s=np.$
$s, t, r,$	$p=\frac{s}{n}.$
$s, r, p,$ / $s, t, p,$	$\frac{s}{p}=n.$

The rate r being given, t the time is found oppoſite to n; and t being given, r is found above n in the tables.

Ex. 1ſt, Sought, the amount, L. 356, 10 s. for 160 days, at 4 per cent. By Table I. np=1.017533 ×356.5=L. 362.75; and intereſt=L. 6.25

2d, Sought, the amount of L. 300 for 7 years and 165 days, at five per cent. By Table II. and I. $\overline{n+n-1} \times p = 1.3726 \times 300 =$ L. 411.78. amount ſought.

3d,

3d, Sought, the time in which L. 300 will amount to L. 400 at 5 per cent. Here $\frac{r}{p}=\frac{400}{300}=$ 1.33333=6 years +200+40+3 days.

TABLE III. Shewing the present Value of L. 1 for Days, at five per cent.

Dys		1	2	3	4	5	6	7	8	9
0	.99	9863	9726	9589	9452	9315	9179	9042	8905	8768
10	8632	8495	8359	8277	8086	7950	7815	7678	7541	7403
20	7266	7128	6991	6857	6723	6587	6451	6365	6179	6043
30	5907	5771	5635	5549	5463	5377	5291	5106	4921	4735
40	4550	4414	4279	4144	4009	3873	3738	3603	3468	3332
50	3197	3062	2927	2792	2657	2522	2388	2253	2118	1983
60	1848	1713	1578	1443	1309	1174	1040	0905	0771	0636
70	0502	0367	0233	0099	9965	9831	9697	9563	9430	9296
80	.989163	9028	8893	8758	8623	8489	8356	8223	8090	7956
90	7823	7689	7556	7422	7288	7154	7021	6887	6754	6621
100	6488	6356	6225	6093	5962	5827	5692	5557	5422	5289
110	5156	5023	4891	4758	4625	4490	4360	4227	4094	3961
120	3829	3696	3564	3431	3299	3166	3034	2901	2769	2686
130	2504	2372	2240	2107	1975	1843	1711	1579	1447	1315
140	1184	1052	0920	0788	0657	0525	0394	0262	0131	9999
150	.979868	9736	9605	9473	9342	9210	9079	8947	8816	8684
160	8553	8422	8291	8160	8029	7898	7767	7636	7505	7374
170	7243	7112	6982	6851	6720	6589	6449	6318	6187	6057
180	5936	5806	5675	5545	5415	5284	5154	5024	4893	4763
190	4633	4503	4373	4243	4113	3983	3853	3723	3593	3463
200	3333	3203	3074	2944	2815	2685	2556	2426	2297	2167
210	2038	1909	1779	1650	1521	1391	1262	1133	1003	0874
220	0745	0616	0487	0358	0229	0101	9972	9843	9714	9585
230	.969457	9328	9200	9071	8942	8814	8685	8557	8428	8309
240	8171	8043	7914	7786	7658	7529	7401	7273	7144	7016
250	6888	6760	6632	6504	6376	6248	6121	5993	5865	5737
260	5609	5481	5354	5226	5098	4971	4843	4716	4588	4460
270	4333	4205	4078	3951	3824	3697	3570	3442	3315	3188
280	3061	2934	2807	2680	2553	2426	2300	2173	2046	1919
290	1792	1665	1539	1412	1286	1159	1033	0906	0770	0653
300	0527	0401	0274	0148	0022	9896	9769	9643	9517	9391
310	.959265	9139	9013	8887	8761	8635	8509	8384	8258	8132
320	8006	7880	7755	7629	7503	7378	7252	7127	7001	6875
330	6750	6625	6499	6374	6249	6124	5999	5873	5748	5623
340	5498	5373	5248	5123	4998	4873	4748	4624	4499	4374
350	4249	4124	3999	3875	3750	3625	3501	3376	3251	3127
360	3002	2878	2753	2629	2505	2381				

TABLE

TABLE IV. Shewing the present Value of L. 1 for Years.

Ys	4 pr cent.	5 pr cent.	6 pr cent.	Ys	4 pr cent.	5 pr cent.	6 pr cent.
1	.961538	.952381	.943396	26	.490196	.434781	.390625
2	.925926	.909091	.892857	27	.480769	.425532	.381679
3	.892857	.869565	.847457	28	.471698	.416666	.373134
4	.862069	.833333	.806451	29	.462963	.408163	.364963
5	.833333	.8	.769230	30	.454545	.4	.357143
6	.806451	.769230	.735294	31	.446428	.393157	.349650
7	.781250	.740740	.704225	32	.438596	.384615	.342466
8	.757576	.714286	.675675	33	.431034	.377358	.335570
9	.735294	.689655	.649350	34	.422728	.370370	.328947
10	.714286	.666666	.625	35	.416666	.363636	.322581
11	.694444	.645161	.602409	36	.409836	.357143	.316456
12	.675675	.625	.581395	37	.403226	.350877	.310559
13	.657895	.606060	.561797	38	.396825	.344827	.304878
14	.641025	.588235	.543478	39	.390625	.338983	.299401
15	.625	.571428	.526315	40	.384615	.333333	.294117
16	.609756	.555555	.510204	41	.378788	.327870	.289017
17	.595238	.540540	.495049	42	.373134	.322581	.284091
18	.581395	.526315	.480769	43	.367647	.317460	.279330
19	.568182	.512820	.467289	44	.362319	.3125	.274725
20	.555555	.5	.454545	45	.357143	.307692	.270270
21	.543478	.487804	.442477	46	.352113	.303030	.265957
22	.581915	.476190	.431034	47	.347222	.298509	.262383
23	.520833	.465116	.420168	48	.342466	.294117	.257732
24	.510204	.454545	.409836	49	.337838	.289855	.253807
25	.5	.444444	.4	50	.33333	.285714	.25

Tables III. and IV. shew the present value of L. 1 for days or years, and are constructed by Equation 2d, 13. whereby n, the tabular number, $=\frac{1}{1+tr}$; or they are the reciprocals of Table I. and II. whereby $\frac{1}{n}$ of Table I. and II.$=n$ of Table III. and IV. From the nature of these two Tables, $1 : n :: s : ns=p$, the principal or present worth of s, for the time and at the rate corresponding to n.

When the time of diſcount conſiſts of years and days, in this caſe, $n \times ns = p$, the preſent worth. Hence the following

TABLE.

Given.	
s, t, r,	$p = ns$.
p, t, r,	$s = \frac{p}{n}$
s, r, p, } s, t, p, }	$n = \frac{p}{s}$

Ex. 1ſt, What ready money will take up a bill of L. 685, 12 s. due 255 days hence, intereſt being reckoned at 5 per cent? By Table III $ns = .966248 \times 685.6 = $ L. $662.4596 = p$, and diſcount = L. 23.14.

2d, Required the preſent worth of L. 250, diſcounting for 10 years, at the rate of 5 per cent. By Table IV, $ns = .666\beta \times 250 = $ L. $166.666 = p$, and diſcount = L. 83.33.

3d, What ready money will pay a debt of L. 150 due 5 years and 95 days hence, intereſt being reckoned at 5 per cent.? By Table IV. and III. $nns = .8 \times .987154 \times 150 = $ L. $118.45848 = p$, and L. $31.541 =$ diſcount.

4th, Having s, p, and r, in the laſt example, ſought, t the time. $\frac{t}{t} = \frac{118.45848}{150} = .7897232 = nn$, and $\frac{.7897232}{.8 = 5 \text{ yrs}} = .987154 = 95$ days.

Theſe examples may with equal facility be determined by the equations annexed to Table I. and II.

Re-

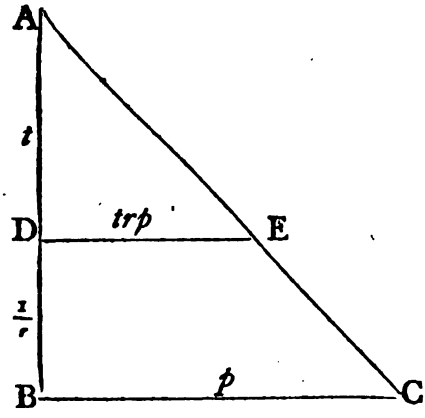
Remarks upon Simple Intereſt.

23. 1ſt, If the time be ſought in which a ſum of money will double itſelf at a given rate of intereſt. In this caſe, the intereſt being equal to the principal, $trp=p$, which gives $tr=1$, $t=\frac{1}{r}$ and $r=\frac{1}{t}$; ſo that t and r are reciprocal of and diſcover one another, when a ſum of money doubles itſelf at ſimple intereſt. Or thus, ſeeing (12) intereſt$=\frac{tp}{20}$, or $\frac{tp}{25}$, &c. according to the ſeveral rates of intereſt; hence if $t=20$, 25, &c. when the rates are 5, 4, &c. per cent. the intereſt will equal the principal. So that

Intereſt$=p$, when $t=\frac{1}{r}=$	33·33	3
	28.57	$3\frac{1}{2}$
	25	4
	22.22 years at	$4\frac{1}{2}$ per cent. per ann.
	20	5
	16.66	6
	12.5	8
	10	10, &c.

2d, The ſimple intereſt of any ſum of money is, *ceteris paribus*, in the direct proportion of p, t, or r; ſo that doubling p, while t and r remain the ſame, the intereſt will be doubled alſo, &c. For as intereſt$=trp$, if you increaſe or diminiſh any one of theſe three factors, the intereſt will be increaſed or diminiſhed in the ſame proportion.

3d, The intereſt of any ſum of money, for a given time, and at a given rate, may be diſcovered by means of a right-angled triangle. Thus.



In conſtructing the right-angled triangle ABC, make AB=$\frac{1}{r}$, and BC=p; in AB take the point D, above or below BC, making AD=t, and join AC; then DE drawn parallel to BC will repreſent the intereſt ſought. For AB, or $\frac{1}{r}$ being equal to the time in which p will double itſelf at the rate r, BC=intereſt of p at the end of that time; but AB : BC :: AD : DE, therefore, by the laſt remark, DE=intereſt of p for the time t, and at the rate r.

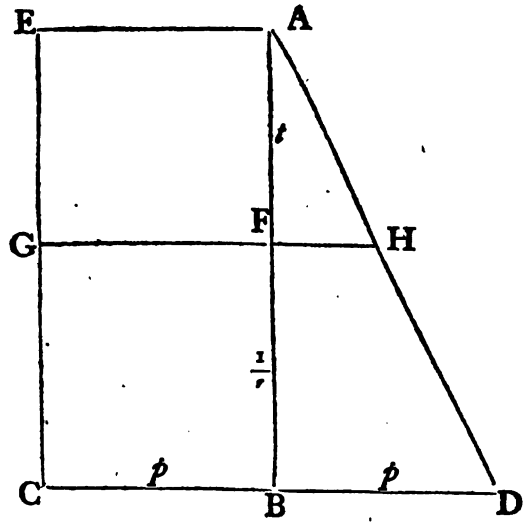
Ex. Suppoſe p=L. 460, t=12 years, and the rate 5 per cent.; then AB=20, BC=460, AD=12, and DE=L. 276=intereſt.

4th, The different amounts are, *ceteris paribus*, in the direct proportion of their principals or preſent worths; ſo that doubling p, while t and r remain the ſame, the amount s will be doubled alſo, &c. For as the amount $s=p+trp$, ſo $2s=2p+tr\times 2p$.

But the different amounts of the ſame principal, or the different preſent worths of the ſame bill, are not in the direct proportion of t or r; for $2s$ is greater than $p+2trp$; and though $\frac{s}{1+tr}=p$, yet

yet $\frac{s}{1+2r}$ is greater than $\frac{1}{2}p$. Hence the different diſcounts of the ſame bill are not in the direct proportion of the times or rates.

5th, The amount of any ſum of money, for a given time, and at a given rate, may be diſcovered by a geometrical figure, two of whoſe oppoſite ſides are parallel, and two of its angles right. Thus, in conſtructing the annexed figure, make



AB=$\frac{1}{r}$, CD, biſected in B, = $2p$; in AB take AF =t; and having completed the parallelogram AC and triangle ABD, GH drawn through F parallel to CD will repreſent the amount ſought. For as GF=p, and FH, by remark 3d, is equal to trp, hence GH=s the amount.

Ex. Suppoſe p=L. 300, t=9 years, and the rate 5 per cent.; then AB=20, AF=9, CD=600, GF =300, and GH=L. 435, the amount.

6th, As in ſimple intereſt, ſtrictly ſpeaking, the intereſt is ſuppoſed not to be uplifted while the

the principal remains in the borrower's hands; it muſt alter the caſe greatly when that intereſt is paid yearly, and make it partake of the nature of compound intereſt, ſeeing one hath the uſe, and can diſpoſe of that intereſt.

Supplement to Chapter II.

1ſt, Of intereſt due upon caſh-accounts and partial payments.

24. In calculating intereſt on caſh-accounts, or where ſeveral partial payments are made at different periods on the ſame ſum, multiply the principal and the ſeveral balances into the number of days they are at intereſt; the ſum of theſe products divided by 7300 (20) will give the total intereſt at 5 per cent.

Ex. 1. Lent A B, the 11th of November 1788, L. 800, which I received in the following partial payments; what intereſt is due at 5 per cent?

		L.	d	
1788. Nov. 11.	Principal lent, -	800	65	52000
1789. Jan. 15.	Received in part, -	250		
	Balance,	550	92	50600
April 17.	Received in part, -	150		
	Balance,	400	62	24800
June 18.	Received in part, -	100		
	Balance,	300	62	18600
Aug. 19.	Received in part, -	125		
	Balance,	175	37	6475
Sept. 25.	Received in full of principal,	175		
		0		1524.75=20.887 =Int.

73

Bankers

Bankers ſometimes receive at 4 and lend at 5 per cent. In this caſe it may be proper to conſider the money lent by the banker as debtor, and the money received by him as creditor, and to make two columns for products, where the intereſt ariſing from the debtor products is to be computed at five, aad the other at 4 per cent.; the difference of theſe two is the balance of intereſt due to or by the banker.

Ex. 2. A B runs a caſh-account with a banker to the extent of L. 500, at 5 per cent. for the balances due by him, and has allowed him 4 per cent. for ſuch balances as may be due to him. What intereſt will be due, and by whom, in conſequence of the following tranſactions?

1789.		L.	d.	Dr	Cr
Jan. 8.	Dr	200	61	12200	
March 10.	Cr	350			
	Cr	150	66		9900
May 15.	Dr	310			
	Dr	160	66	10560	
July 20.	Cr	240			
	Cr	80	52		4160
Sept. 10.	Dr	410			
	Dr	330	52	17160	
Nov. 1.	Cr	420			
	Cr	90	49		4410
Dec. 20.	Dr	250			
1790.					
Jan. 8.	Dr	160	19	3040	
				42960	18470

$\frac{429.60}{73} - \frac{4}{5}) \frac{184.7}{73}$ = L. 3.86 Int. } due to the banker.
160.00 Prin. }

When partial payments are made on bills or bonds at any interval greater than a year, the legal and uſual method is to add the intereſt at the times of payment to the principal, and from that amount to deduct the payment.

Ex. 3d. Borrowed on bond, June 1. 1781, the ſum of L. 1000, at 5 per cent. and made partial payments as follows; required, the ſtate of the affair on the 14th of Auguſt 1785.

		L.
1781. June 1.	Principal borrowed,	1000
	Intereſt for 1 year and 129 days, -	67.671
1782.	Amount,	1067.671
Oct. 8.	Paid in part, -	250
	Balance,	817.671
	Intereſt for 1 year and 85 days, -	50.4
1784.	Amount,	868.071
Jan. 1.	Paid in part, -	408.071
	Balance,	460.
	Intereſt for 1 year and 225 days, -	37.178
1785.	Amount,	497.178
Aug. 14.	Paid in full, -	497.178

2. Of

2d. Of the equation of payment.

25. When two or more debts are payable at different times; the finding a mean time at which all the debts may be paid at once, without loſs to debtor or creditor, is called equating the terms of payment, and may be performed by the following

RULE.

Multiply the ſeveral debts into their reſpective times, and divide the ſum of the products by the total debts; the quotient is accounted the mean time.

Ex. A owes B L. 800, whereof L. 200 are to be paid at the end of 2 months, L. 300 at the end of 6 months, and L. 300 at the end of 10 months; required the equated time for paying the whole.

$$\begin{array}{rl} 200 \times 2 = & 400 \\ 300 \times 6 = & 1800 \\ 300 \times 10 = & 3000 \\ \hline 800 &)5200(6.5 \text{ months, anſwer.} \end{array}$$

The above method is eaſy, but not accurate: for a perſon, by keeping money unpaid after it becomes due, gains the intereſt thereof for that time; but by paying money before it is due, he does not loſe the intereſt, as the Rule ſuppoſes, but only the diſcount thereof for that time, which is always leſs than the intereſt, by the intereſt of the diſcount for the time; therefore take the following

RULE.

Find, by Equation 2d, 13. the preſent worth of each debt, and by Equation 3d, find in what time the ſum of the preſent worths will amount to the ſum of the debts.

Ex. A owes B L. 250, whereof L. 50 is payable 1 year hence, L. 100 3 years hence, and L. 100 5 years hence; what is the equated time for paying all theſe debts at once, intereſt reckoned at 5 per cent.? Here,

$\frac{50}{1.05} = 47.619$ then $\frac{r-p}{rp} = \frac{35.425}{10.728} = 3.302 = 3$ years

$\frac{100}{1.15} = 86.956$ and 110 days.

$\frac{100}{1.25} = \frac{80.000}{214.575} = p.$

3d. Of the premium of commiſſion.

It is uſual with bankers, in granting a draught upon their correſpondents either at home or abroad, payable to the bearer or order, for value received, to charge ſo much per cent. upon the tranſaction, in name of commiſſion; which commiſſion varies from $\frac{1}{8}$ to $2\frac{1}{2}$ per cent. according to the nature of the caſe; hence the following

TABLE.

Per cent.				Com. on L. 1.	Per cent.			Com. on L. 1.
	L.	s.	d.			L.	s.	
$\frac{1}{8}$	0	2	6	.00125	$1\frac{1}{4}$	1	5	.0125
	0	3		.0015	$1\frac{1}{2}$	1	10	.015
$\frac{1}{6}$	0	3	4	.0016β	$1\frac{3}{4}$	1	15	.0175
$\frac{1}{5}$	0	4		.002	2	2	0	.02
$\frac{1}{4}$	0	5		.0025	$2\frac{1}{4}$	2	5	.0225
$\frac{1}{3}$	0	6	8	.003β	$2\frac{1}{2}$	2	10	.025
$\frac{1}{2}$	0	10		.005	$2\frac{3}{4}$	2	15	.0275
$\frac{3}{4}$	0	15		.0075	3	3	0	.03
1	1	0		.01	$3\frac{1}{2}$	3	10	.035

Ex. Sought, the commiſſion upon L. 542 at $1\frac{1}{2}$ per cent. Here 542x.015=L. 8.13, the commiſſion.

CHAP.

CHAP. III.

Of ANNUITIES *computed at* SIMPLE INTEREST.

26. AN annuity, rent, or penſion, is a ſum of money payable yearly, every half year, or quarterly, to continue for a certain number of years, for life, or for ever. When an annuity continues unpaid after it falls due, it is then ſaid to be in arrears. When the purchaſer does not immediately enter upon poſſeſſion, the annuity not commencing till ſome time after, it is ſaid to be in reverſion; and the reverſion of an annuity is the preſent value of a perpetuity which is to commence after the expiration of the annuity, being the ſame with a reverſion in fee-ſimple.

1ſt. Of the amount of an annuity in arrears.

27. In computing annuities at ſimple intereſt, let,

a repreſent the annuity, rent, or penſion.
p its preſent value.
t the time of its continuance.
n the number of years before it commences.
r the intereſt of L. 1 for one year.
s the amount of an annuity and its intereſt.

Then, ſince L. 1 : r : : a : ar, and as the firſt year's annuity bears no intereſt, being due only at the end of the year, we ſhall have, from an yearly annuity, a,

$a \times \overline{1+0}$ = amount of the 1ſt year.
$a \times \overline{1+r}$ = amount of the 2d year.
$a \times \overline{1+2r}$ = amount of the 3d year.
$a \times \overline{1+3r}$ = amount of the 4th year.
$a \times \overline{1+\overline{t-1}\times r}$ = amount of the t year.

Here the number of terms of this ſeries being t, the ſum $s = ta \times 1 + \overline{\frac{t-1}{2}r}$ (8). Hence the following

TABLE.

Given.		
$a, t, r,$	$s = t\,a \times \overline{1+\frac{t-1}{2}r} = ta + \frac{t}{2} \times \overline{tra-ra}$.	Equation 1.
$s, t, r,$	$a = \frac{s}{t \times 1+\frac{t-1}{2}r}$,	2.
$s, t, a,$	$r = \frac{\frac{2s}{t}-2a}{\overline{t-1}\times a}$	3.
$s, r, a,$	$t = \sqrt{\frac{2s}{ar}+\frac{1}{r}-\frac{1}{2}}^2 - \overline{\frac{1}{r}-\frac{1}{2}}$	4.

Sought r, in Equation 3d.

Equ. 1. $s = ta + \frac{t^2a-ta}{2}r$.
$2s - 2ta = \overline{t^2a-ta} \times r$.
$\frac{2s}{t} - 2a = \overline{t-1} \times ar$.

Sought t, in Equation 4th.

Equ. 1. $2s = 2at + \overline{t^2a-ta} \times r$.
$\frac{2s}{r} = \frac{2at}{r} + t^2a - ta$.
$t^2 + \frac{2t}{r} - t = \frac{2s}{ar}$.
$t + \overline{\frac{1}{r}-\frac{1}{2}} = \sqrt{\frac{2s}{ar} + \overline{\frac{1}{r}-\frac{1}{2}}^2}$.

Ex. 1ſt.

Ex. 1st, Required the amount of an annuity of L. 20 per annum, being in arrears for 12 years, at 5 per cent. simple interest. Here $a=20$, $t=12$, $r=.05$, and $ta\times\overline{1+\frac{t-1}{2}r}=240\times1.275=306=s$.

2d, Sought, the annuity, which being in arrears for 9 years will amount to L. 250, at 5 per cent. Here $s=250$, $t=9$, $r=.05$.

$\frac{s}{t\times1+\frac{t-1}{2}r}=\frac{250}{10.8}=$L. 23.148 = annuity.

3d, Sought, the rate of interest at which an annuity of L. 20 per annum, being in arrears for 12 years, will amount to L. 306.

$\frac{\frac{2s}{t}-2a}{t-1\times a}=\frac{11}{220}=.05=5$ per cent.

4th, In what time will an yearly annuity of L. 85.5 amount to L. 2522.25, at the rate of 5 per cent. per annum?

$\sqrt{\overline{\frac{2s}{a}+\frac{1}{r}-\frac{1}{2}}^{2}}-\frac{1}{r}-\frac{1}{2}=\sqrt{1180+380.25}-19.5=20$ years.

28. When an annuity is paid every half year, quarterly, or monthly; then for half years payments, take half the yearly annuity, half of the ratio, and twice the number of years; and for quarterly payments, take the fourth part of the yearly annuity and of the ratio, and four times the number of years, &c. and work as directed above.

To Extract the Square Root of any Quantity.

Let the given quantity $=g$, and its square root $=r+e$, then

$rr+2re+ee=g$

$2re+ee=g-rr=d$

$\frac{d}{2r+e}=e$.

Example.

Let $g=$ 1560.25 (39.5 $=r+e$,

$rr=$ 9

$2r+e=69)660=d$

621

$2r+e=785)3925=d$

3925

Ex. What

Ex. What will an yearly annuity of L. 70, payable every quarter of a year, amount to in 5 years, at 5 per cent. Here $a=17.5$, $t=20$, $r=.0125$, and $350\times1.11875=391.56=s$.

2d, Of the preſent worth of an annuity.

29. When an annuity is to continue for a certain number of years, its preſent value p, at a given rate of ſimple intereſt, may be found thus: Since $1+tr$, the amount of L. 1 for any time, hath the ſame ratio to L. 1, as $ta\times\overline{1+\frac{t-1}{2}r}$, the amount of any annuity a, hath to p its preſent value; therefore $\overline{1+tr}\times p=ta\times\overline{1+\frac{t-1}{2}r}$, from which all the varieties in this caſe will be reſolved, as in the following

TABLE.

Given.		
$a, r, t,$	$p=ta\times\frac{1+\frac{t-1}{2}r}{1+tr}$	Equ. 1.
$p, r, t,$	$a=\frac{p}{t}\times\frac{1+tr}{1+\frac{t-1}{2}r}$	2.
$p, t, a,$	$r=\frac{\frac{p}{t}-a}{\frac{t-1}{2}a-p}$	3.
$p, r, a,$	$t=\sqrt{\frac{2p}{ar}+\frac{1}{r}-\frac{p}{a}-\frac{1}{2}}^2-\frac{1}{r}-\frac{p}{a}-\frac{1}{2}$	4.

Sought r in Equation 3d.

Equ. 1. $p+trp=ta+\frac{at^2-at}{2}r.$

$\frac{p}{t}+rp=a+\frac{t-1}{2}ar.$

$\frac{p}{t}-a=\frac{t-1}{2}ar-rp.$

Sought t in Equation 4.

Equ. 1. $2p+2rpt=2at+\overline{t^2-1}\times ar.$

$\frac{2p}{ar}+\frac{2p}{a}t=\frac{2}{r}t+t^2-t.$

$t^2-\frac{2p}{a}\left\{\begin{matrix}+\frac{2}{r}\\-1\end{matrix}\right\}t=\frac{2p}{ar}.$

Ex. 1ſt.

Ex. 1. Required the preſent value of a yearly annuity of L. 20, to continue 12 years, allowing diſcount at 5 per cent. ſimple intereſt. Here,

$$ta \times \frac{1+\frac{t-1}{2}r}{1+tr} = 240 \times \frac{1.275}{1.6} = \text{L. } 191.2488.$$

2. A perſon is willing to lay out the ſum of L. 300 in the purchaſe of an yearly annuity, to continue 15 years, being allowed diſcount at 6 per cent; ſought the annuity. Here $\frac{p}{t} \times \frac{1+tr}{1+\frac{t-1}{2}r} = 20 \times 1.338 = \text{L. } 26.76 = \text{annuity.}$

3. There is an annuity of L. 50 a-year, to continue 20 years, to be ſold for L. 737.5, ready money; ſought, the rate of ſimple intereſt at which the diſcount is given.

$$\frac{\frac{p}{t}-a}{\frac{t-1}{2}a-p} = \frac{-13.125}{-262.5} = .05 = 5 \text{ per cent.}$$

4. There is an yearly annuity of L. 30, to continue a certain number of years, which can be ſold for L. 405, diſcounting at 5 per cent; ſought, the time of its continuance. $\sqrt{540+36}-6 = 18$ years.

30. When an annuity is paid every half year, quarterly, or monthly, and its preſent value be ſought. See 28.

Ex. What will be the preſent value of an yearly annuity of L. 70, payable every quarter of a year, and to continue 5 years, diſcounting intereſt at 5 per cent. Here $a=17.5$, $t=20$, $r=.0125$, and $350 \times .895 = 313.25 = p$.

In like manner a, t, or r may be found.

3d, Of

3d. Of the preſent worth of an annuity in reverſion.

31. When an annuity is to commence after a certain number of years (n), and then to continue for a certain time (t), its preſent value may be found thus: Find its amount at the end of the time t, (Equation 1. 27.) and divide this amount by $1+t+nr$, the quotient will give p, the preſent value. Thus $ta \times \frac{1+\frac{t-1}{2}r}{1+t+nr} = p$.

Ex. Required the preſent value of an yearly annuity of L. 30, which is to commence 6 years hence and to continue 15 years, diſcounting intereſt at 5 per cent. Here $a=30$, $t=15$, $n=6$, $r=.05$, and $450 \times \frac{1.35}{2.05} =$ L. 296.3 $= p$.

4th. Of the preſent worth of the reverſion of an annuity.

32. The preſent value of the reverſion of an annuity, being the ſame with that of a perpetuity which is to commence after a certain number of years (n), may be found thus: From the value of the perpetuity (ſee Remark 3d), ſubtract that of the annuity, the remainder gives the preſent value of the reverſion: Or thus, divide the value of the perpetuity, viz. $\frac{a}{r}$ by $1+nr$, the quotient gives the preſent value of the reverſion, thus $\frac{\frac{a}{r}}{1+nr} = p$.

Ex. Sought, the reverſion of an yearly annuity of L. 20, which is to continue 12 years, allowing diſcount at 5 per cent. Here

$$\frac{\frac{a}{r}}{1+nr} = \frac{20}{.08} = \text{L. } 250 \text{ the reverſion.}$$

33. The

33. The ſeveral particulars reſpecting annuities at ſimple intereſt may be diſcovered by means of the two following Tables, wherein are calculated the amount and preſent value of L. 1 annuity for years, at ſeveral rates of intereſt.

TABLE V. Shews the Amount of L. 1 Annuity for Years.

Yrs	4 pr cent.	5 pr cent.	6 pr cent.	Yrs	4 pr cent.	5 pr cent.	6 pr cent.
1	1.00	1.00	1.00	26	39.00	42.25	45.50
2	2.04	2.05	2.06	27	41.04	44.55	48.06
3	3.12	3.15	3.18	28	43.12	46.90	50.68
4	4.24	4.30	4.36	29	45.24	49.30	53.36
5	5.40	5.50	5.60	30	47.40	51.75	56.10
6	6.60	6.75	6.90	31	49.60	54.25	58.90
7	7.84	8.05	8.26	32	51.84	56.80	61.76
8	9.12	9.40	9.68	33	54.12	59.40	64.68
9	10.44	10.80	11.16	34	56.44	62.05	67.66
10	11.80	12.25	12.70	35	58.80	64.75	70.70
11	13.20	13.75	14.30	36	61.20	67.50	73.80
12	14.64	15.30	15.96	37	63.64	70.30	76.96
13	16.12	16.90	17.68	38	66.12	73.15	80.18
14	17.64	18.55	19.46	39	68.64	76.05	83.46
15	19.20	20.25	21.30	40	71.20	79.00	86.80
16	20.80	22.00	23.20	41	73.80	82.00	90.20
17	22.44	23.80	25.16	42	76.44	85.05	93.66
18	24.12	25.65	27.18	43	79.12	88.15	97.18
19	25.84	27.55	29.26	44	81.84	91.30	100.76
20	27.60	29.50	31.40	45	84.60	94.50	104.40
21	29.40	31.50	33.60	46	87.40	97.75	108.10
22	31.24	33.55	35.86	47	90.24	101.05	111.86
23	33.12	35.65	38.18	48	93.12	104.40	115.68
24	35.04	37.80	40.56	49	96.04	107.80	119.56
25	37.00	40.00	43.00	50	99.00	111.25	123.50

Table V. ſhews the amount of L, 1 annuity, and is conſtructed by equation 1. 27. whereby n the tabular number $= t \times 1 + \frac{t-1}{2} r$; Or thus, ſeeing

$1 \quad = a$ the amount for 1 year,
$a + 1 + r = b$ for 2 years,
$b + 1 + 2r = c$ for 3 years,
$c + 1 + 3r = d$ for 4 years, &c.

Hence, to L 1, the firſt year of this Table, add the firſt year of Table II. their ſum is the ſecond year of this Table; to which add the ſecond year of Table II. their ſum is the third year of this Table, &c. From the nature of the Table, $1 : n :: a : na = s$ the amount of any annuity a, for the time and at the rate correſponding to n; hence the following

TABLE.

Given.	
$a, t, r,$	$s = na.$
$s, t, r,$	$a = \frac{s}{n}.$
$s, t, a,$ / $s, r, a,$	$n = \frac{s}{a}.$

Ex. 1. what will an yearly annuity of L. 35 amount to in 16 years, at the rate of 5 per cent. ſimple intereſt? $na = 22 \times 35 =$ L. 770, the amount.

2. What yearly annuity will amount to L. 471 in the ſpace of 20 years, at 6 per cent. ſimple intereſt? $\frac{s}{n} = \frac{471}{31\cdot4} =$ L. 15, the annuity.

TABLE

TABLE VI. Shews the preſent Value of L. 1 Annuity for Years.

Yrs	4 per cent.	5 per cent.	6 per cent.	Yrs	4 per cent.	5 per cent.	6 per cent.
1	0.96154	0.95238	0.94339	26	19.11764	18.36956	17.77343
2	1.88888	1.86363	1.83928	27	19.71154	18.95744	18.34351
3	2.78571	2.73913	2.69491	28	20.33962	19.54166	18.91044
4	3.65517	3.58333	3.51613	29	20.92592	20.12245	19.47262
5	4.5	4.4	4.30769	30	21.54545	20.7	20.03571
6	5.32258	5.19231	5.07353	31	22.14285	21.27451	20.59440
7	6.125	5.96296	5.81690	32	22.73688	21.84615	21.15067
8	6.90909	6.71428	6.54054	33	23.32758	22.41509	21.70469
9	7.67647	7.44827	7.24675	34	23.91525	22.98148	22.35526
10	8.42857	8.16666	7.9375	35	24.5	23.54545	22.80645
11	9.16666	8.87096	8.61445	36	25.08197	24.10714	23.35443
12	9.89189	9.5625	9.27907	37	25.66129	24.66666	23.90062
13	10.60526	10.24242	9.93258	38	26.23809	25.22414	24.44512
14	11.30769	10.91176	10.85326	39	26.8125	25.77965	24.98802
15	12.	11.57143	11.21052	40	27.38461	26.33333	25.52941
16	12.68392	12.22222	11.83673	41	27.95454	26.88524	26.06936
17	13.35714	12.86486	12.45544	42	28.52238	27.43548	26.60795
18	14.02325	13.5	13.06706	43	29.08823	27.98412	27.14525
19	14.68182	14.12820	13.67289	44	29.65217	28.53125	27.68132
20	15.33333	14.75	14.27272	45	30.21428	29.07692	28.21621
21	15.97826	15.36585	14.86504	46	30.77465	29.62121	28.75
22	16.61702	15.97619	15.45689	47	31.33333	30.16418	29.28272
23	17.24948	16.58139	16.04201	48	31.89041	30.70588	29.81444
24	17.87755	17.18182	16.60246	49	32.44594	31.24638	30.34518
25	18.5	17.77777	17.2	50	33.	31.78571	30.875

Table VI. ſhews the preſent value of L. 1 annuity, and is conſtructed by Equation 1ſt, 29. whereby n the tabular number $= t \times \frac{1+\frac{t-1}{2}r}{1+tr}$; Or thus, n Table VI. $= \frac{n, \text{T. V.}}{n, \text{T. II.}}$.

This Table is conſtructed upon the ſuppoſition, that the preſent value of an annuity is equal to that of the amount of the ſeveral annual payments, with their intereſt at the expiration of the annuity. From the nature of the Table,

$1 : n :: a : na = p$, the preſent worth of any annuity a, for the time and at the rate correſponding to n. Hence the following

TABLE.

Given.	
$a, r, t,$	$p = na.$
$p, r, t,$	$a = \frac{p}{n}.$
$p, t, a,$	$n = \frac{p}{a}.$
$p, r, a,$	

Ex. 1ſt. What is the preſent worth of an yearly annuity of L. 50, to continue 15 years, diſcounting intereſt at 5 per cent?

$$na = 11.57143 \times 50 = \text{L. } 578.5715 = p.$$

2d. A perſon is willing to lay out the ſum of L. 300 in the purchaſe of an annuity to continue 18 years, being allowed diſcount at 5 per cent.; ſought the annuity.

$$\frac{p}{n} = \frac{300}{13.5} = \text{L. } 22.222 = \text{annuity.}$$

3d. A gentleman hath L. 160, which he would lay out in the purchaſe of an annuity of L. 20 per annum; ſought, how many years the ſaid annuity muſt continue, diſcounting intereſt at 6 per cent.

$\frac{p}{a} = \frac{160}{20} = 8 = n.$ the next leſs to which below 6 per cent. is 7.9375, correſponding to 10 years, and for the odd days ſay

8.61445	8.0000
7.93750	7.9375

as .67695 : 1 :: .0625 : .09233, correſponding to 34 days, (16).

4th. A

4th. A hath an annuity of L. 20 per annum, to continue 7 years; B hath one of L. 5, 10 s. to continue 21 years; ſought, which of theſe is moſt valuable, diſcounting intereſt at 6 per cent.

A's annuity = 5.8169 × 20 = 116.3380
B's annuity = 14.865 × 5.5 = 81.7575

A — B = L. 34.5805

5th. A lends to B L. 360, upon a mortgage of land, whoſe rent is L. 75 per annum; B keeps the money 5 years, during which time A receives the ſaid rent; ſought, the ſtate of their affairs, diſcounting intereſt at 6 per cent.

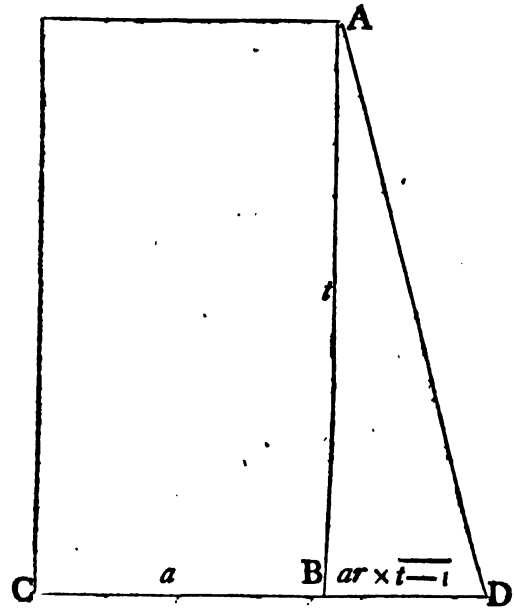
By Table II. 360 × 1.3 = L. 468
Table V. 75 × 5.6 = 420

Balance due by B to A = L. 48

Remarks upon Annuities at Simple Intereſt.

34. 1ſt, The amount of an annuity for a given time, and at a given rate of ſimple intereſt, may be expreſſed by the area of a rectangle and triangle.

Thus,



Thus, in conſtructing the annexed figure, make AB=t, BC, raiſed at right angles to AB, $= a$, and BD = intereſt of a for the time t—1; laſtly, join AD, and complete the rectangle AC; then will AC + ABD expreſs the amount of the annuity a, for the time t, at a given rate of intereſt; for AC + ABD $= at + \frac{t}{2} \times \overline{tru - ra}$ = amount of the annuity.

Ex. Suppoſe a = L. 20, t = 12 years, and the rate 5 per cent.; ſought the amount.

Here AB = 12, BC = 20, BD = 11. Hence, AC + ABD = 240 + 66 = L. 306 = amount.

2d. The amount of an annuity is, *cæteris paribus*, always leſs, and its preſent value greater, upon the principles of ſimple than of compound intereſt, as is manifeſt from inſpecting the Tables; and

and as the tabular numbers expreſſing the preſent worth of an annuity may be conſidered as the years purchaſe of ſaid annuity for the correſponding years, theſe at ſimple intereſt are unlimited. Thus an annuity to continue 99 years, at five per cent. is valued, Equation 1ſt, 29. at 57.4 years purchaſe; whereas the years purchaſe at compound intereſt are always leſs than $\frac{1}{r}$. For theſe reaſons the purchaſe of annuities upon the principles of ſimple intereſt is very unequitable, and compound intereſt, that both parties may employ their money to the greateſt advantage, is uſually admitted in theſe tranſactions.

3d. The value of a perpetuity cannot be obtained upon the principles of ſimple intereſt, becauſe the number of years purchaſe is indefinite. For, as a finite quantity can make no alteration upon an indefinite one, by any operation of arithmetic, the fractional part of the annuity, (Equation 1ſt, 29.) viz. $\frac{1+\frac{t-1}{2}r}{1+tr} = 1$, when t is indefinite; and of conſequence, the number of years purchaſe of a perpetuity, viz. $t \times 1$, is alſo indefinite. Or thus, ſeeing the value of an annuity at ſimple intereſt is ſuppoſed to be equal to the preſent value of the amount of the ſeveral annual payments, with their intereſt, collected into one ſum; but as this, when t is indefinite, cannot be done, therefore the value of a perpetuity upon the principles of ſimple intereſt muſt remain impracticable, and its only value that can be obtained, muſt be derived from the principles of compound intereſt, viz. $\frac{a}{r}$. (56.)

CHAP-

CHAP. IV.

Of GEOMETRICAL PROGRESSION.

35. WHEN a ſeries of quantities, exceeding two in number, increaſes by the ſame common multiplier, or decreaſes by the ſame common diviſor, the terms of the ſeries form a continued Geometrical Progreſſion; and the common multiplier or diviſor is called the common ratio of the ſeries. Thus,

$a,\ ar,\ ar^2,\ ar^3,\ ar^4 \ldots ar^{n-1} = z.$ an increaſing ſeries.

$z,\ \frac{z}{r}\ \frac{z}{r^2},\ \frac{z}{r^3},\ \frac{z}{r^4} \ldots \frac{z}{r^{n-1}} = a.$ a decreaſing ſeries.

1, 2, 4, 8, 16, 32, 64,
1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, 2 the common { Multiplier. / Diviſor.

36. In a geometrical ſeries, the product of the extremes is equal to the product of any two terms

terms equally diſtant from the extremes; thus $az = ar \times \frac{z}{r}$; and the ſquare of any one term is equal to the rectangle under any two other terms equally diſtant from it; thus, $ar^2 \times ar^2 = ar \times ar^3$.

37. In a geometrical ſeries, let

a repreſent the 1ſt term.
z the laſt term.
n the number of terms.
r the common ratio = quot. of the greater by the leſſer contig. term.
s the ſum of the ſeries, and L, the Logarithm of any quantity.

Then, as r is always increaſing from the 2d term, its exponent in the laſt term will be $n-1$, and $ar^{n-1} = z$.

38. In a geometrical ſeries, the ſum of all the terms wanting the leaſt is equal to the ſum of all the terms wanting the greateſt multiplied by the common ratio.

that is, $s - a = \overline{s - z} \times r$, in an increaſing ſeries; and $s - a = \overline{s - z} \times \frac{1}{r}$, in a decreaſing ſeries.

for $\frac{ar,\ ar^2,\ ar^3 \ldots \quad ar^{n-1} =}{r \times a,\ ar,\ ar^2 \ldots \quad ar^{n-2},}$ Hence,

39. In a continued geometrical progreſſion, if any three of the five quantities, a, z, n, r, and s be given, the other two may be determined, as in the following table:

TABLE.

Given.		
$a, r, n,$	$s = a \times \frac{r^n - 1}{r - 1}, \ldots$	Equation 1.
$a, r, z,$	$s = \frac{rz - a}{r - 1} = \frac{z - a}{r - 1} + z, \ldots$	2.
$a, n, z,$	$s = \frac{z - a}{\sqrt[n-1]{\frac{z}{a}} - 1} + z, \ldots$	3.
$r, z, n,$	$s = \frac{z}{r^n - 1} \times \frac{r^n - 1}{r - 1}, \ldots$	4.
$a, r, n,$	$z = ar^{n-1}, \ldots$	5.
$a, r, s,$	$z = \frac{rs - s + a}{r} = s - \frac{s + a}{r}, \ldots$	6.
$r, n, s,$	$z = r^{n-1} s \times \frac{r - 1}{r^n - 1}, \ldots$	7.

Sought s in Equation 1.

38 $s - a = \overline{s - z} \times r.$

37 $z = ar^{n-1}.$

$s - a = rs - ar^n.$

$s \times \overline{r - 1} = a \times \overline{r^n - 1}.$

Sought s in Equation 3.

37 $ar^{n-1} = z.$

$r^{n-1} = \frac{z}{a}.$

$r = \sqrt[n-1]{\frac{z}{a}}.$ Subſtit. this in Equ. 2.

Sought z in Equation 7.

38. $s - \frac{z}{r^{n-1}} = rs - r^n z.$

$rz - \frac{z}{r^{n-1}} = s \times \overline{r - 1}.$

$zr^n - z = r^{n-1} s \times \overline{r - 1}.$

Given.

Given.		
$r, z, n,$	$a = \frac{z}{r^n - 1}, \ldots$	Equation 8.
$r, z, s,$	$a = rz - s \times \overline{r-1}, \ldots$	9.
$r, n, s,$	$a = s \times \frac{r-1}{r^n - 1}, \ldots$	10.
$a, r, z,$	$n = \frac{L.\frac{z}{a}}{L.r} + 1 = \frac{L.z - L.a}{L.r} + 1, \ldots$	11.
$a, r, s,$	$n = \frac{L.\overline{\overline{r-1} \times \frac{s}{a} + a}}{L.r} = \frac{L.\overline{\overline{r-1} \times s + a} - L.a}{L.r},$	12.
$a, z, s,$	$n = \frac{L.z - L.a}{L.\overline{s-a} - L.\overline{s-z}} + 1, \ldots$	13.
$r, z, s,$	$n = \frac{L.z - \overline{L.rz - s \times \overline{r-1}}}{L.r} + 1, \ldots$	14.

Sought a in Equation 11.

37. $r^{n-1} = \frac{z}{a}$.

$n-1 = \frac{L.\frac{z}{a}}{L.r}$.

Sought a in Equation 12.

38. $s - a = rs - ar^n$.

$ar^n = rs - s + a$.

$r^n = \frac{\overline{r-1} \times s + a}{a}$.

Sought a in Equation 13.

Equ. 15. $r = \frac{s-a}{s-z}$. Subſtit. this in Equ. 11.

Soug ht a in Equation 14.

Equ. 9. $a = rz - s \times \overline{r-1}$. Subſtit. this in Equ. 11.

Given.		
$a, z, s,$	$r = \frac{s-a}{s-z}, \ldots$	Equation 15.
$a, n, s,$	$\frac{s}{a}r - r^n = \frac{s}{a} - 1, \ldots$	16.
$a, n, z,$	$r = \sqrt[n-1]{\frac{n}{a}}, \ldots$	17.
$n, z, s,$	$\frac{s}{s-z}r^{n-1} - r^n = \frac{z}{s-z}, \ldots$	18.

In the above table, the expreſſions belong to an increaſing progreſſion, and may be applied alſo to a decreaſing one, by making the quantities a and z change places every where.

EXAMPLES.

1*ſt*, In the geometrical ſeries, 1, 2, 4, 8, 16, &c. containing ſixteen terms, ſought, the laſt term and the ſum of the ſeries. Here $a=1$, $r=2$, $n=16$. Sought, z and s.

Equation 1. $a \times \frac{r^n-1}{r-1} = 1 \times \frac{65526-1}{1} = 65525 = s.$

L. $r = 0.3010300$
$n = 16$
$4.8164800 =$
$65526 = r^n.$

Equation 5. $ar^{n-1} = 1 \times 32768 = z.$

Sought r in Equation 16.

38. $s-a = rs-ar^n.$

$rs-ar^n = s-a.$

$\frac{s}{a}r - r^n = \frac{s}{a} - 1.$

Sought r in Equation 18.

38. $s - \frac{z}{r^{n-1}} = r - zr.$

$sr^{n-1} - z = s-z \times r^n.$

$\frac{s}{s-z}r^{n-1} = \frac{z}{s-z} + r^n.$

2*d*, A gentleman, who had a daughter married, gave the huſband towards her portion 4 s. promiſing to triple that ſum the firſt day of every month, for nine months after the marriage. The ſum paid on the firſt day of the ninth month was 26244 ſhillings. What was the lady's portion? Here are given a, r, z; ſought s.

Equ. 2. $\frac{rz-a}{r-1}=\frac{78728}{2}=39364$ ſh. $=$ L. 1968, 4 s. $=s$.

3*d*, A gentleman buys a fine houſe, in which were 20 threſholds, and, in name of price, was to lay a farthing on the firſt threſhold, a halfpeny on the ſecond, a penny on the third; doubling the ſum on every following threſhold; what would the houſe coſt him? Here are given, a, r, n; ſought s.

Equ. 1. $a\times\frac{r^n-1}{r-1}=1048575$ Far. $=$ L. 1092, 5 s. 4 d. $=$ Price.

4*th*, A Gentleman ſold his eſtate for L. 21844, which was diſcharged by ſeveral payments in geometrical progreſſion; the firſt was L. 4, the laſt L. 16384; what was the ratio, and how many payments were there? Here are given a, z, s; ſought r, n.

Equation 15. $\frac{s-a}{s-z}=\frac{2184}{546}=4=$ Ratio.

Equation 13. $\frac{L.z-L.a}{L.\overline{s-a}-L.\overline{s-z}}+1=\frac{3.61236}{.60206.}+1=7=n$.

Of

Of a Geometrical Series continued indefinitely.

40. As in a geometrical ſeries $s-a = \overline{s-z} \times r$, (38), hence $1 : r :: s-z : s-a$, and $1 : r-1 :: s-z : z-a$. but as the quantities a, z, change places every where, when applied from an increaſing to a decreaſing ſeries, we ſhall have $1 : r-1 :: s-a : a-z$; therefore ſuppoſing z to vaniſh, or the decreaſing ſeries to have no laſt term, then $1 : r-1 = s-a : a$; which gives $s = \frac{ra}{r-1}$; hence the following

TABLE.

Given.		
a, r,	$s = \frac{ra}{r-1} = ra \times \frac{1}{r-1}$, . . .	Equation 1.
a, s,	$r = \frac{s}{s-a} = s \times \frac{1}{s-a}$, . . .	2.
r, s,	$a = \frac{s \times \overline{r-1}}{r} = s \times \overline{1 - \frac{1}{r}}$, ...	3.

EXAMPLES.

1*ſt*, Required the ſum of the ſeries, $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$, &c.

Here $a = 1$, $r = 2$, and $s = \frac{2 \times 1}{1} = 2$, the ſum.

2*d*, Required the ſum of the ſeries $.333 = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$, &c.

Here $a = \frac{3}{10}$, $r = 10$, and $s = \frac{3 \times 10}{10 \times \overline{10-1}} = \frac{1}{3}$, the ſum.

3*d*, $\frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3}$ &c. $= \frac{R \times \frac{1}{R}}{R-1} = \frac{1}{R-1} = \frac{1}{r}$.

CHAP.

CHAP. V.

Of COMPOUND INTEREST.

41. WHEN the intereſt of money at the end of each year is added to the principal ſum, and both bear intereſt the following year, money is ſaid to be at compound intereſt: In which let

p repreſent any principal ſum lent out.
t the time.
r the intereſt } of L. 1 for one year.
R the amount }
s the amount of principal and intereſt; and L the logarithm of a quantity.

Then L. 1, at the end of the year, will become $1+r=R$, hence the geometrical ſeries $1 : R : R^2 : R^3 : R^4 \ldots\ldots R^t$. So R^t will be the amount of L. 1, for the time t, and at the rate r. But $1 : p :: R^t : pR^t = s$, the amount of p, at the end of t years, at the rate r; hence the following table.

TABLE.

TABLE.

Given.		
$p, R, t,$	$s = pR^t$. or $L.p + L.R \times t = L.s$ . . .	Equa. 1.
$s, R, t,$	$p = \frac{s}{R^t}$ or $L.s - L.R \times t = L.p$, ...	2.
$s, R, p,$	$t = \frac{L.s - L.p}{L.R}$, ...	3.
$s, p, t,$	$R = \sqrt[t]{\frac{s}{p}}$. or $L.R = \frac{L.s - L.p}{t}$, ...	4.

Ex. 1. Required, the amount of L. 25 foreborn 12 years, at 5 per cent. compound intereſt. Here, $p = 25$, $t = 12$, $R = 1.05$.

$L.p = 1.3979400$	$L.R = 0.0211893$
$+ L.R \times t = .2542716$	$t = 12$
Sum $= 1.6522116 = L.\ 44.896$	$.2542716$

2d. Sought, the principal which, in the ſpace of 12 years, will amount to L. 300, reckoning intereſt at 4 per cent. Here,

$L.s = 2.4771213$	$L.\ 1.04 = 0.0170333$
$- L.R \times t = .2044002$	$t = 12$
Diff. $= 2.2727211 = L.\ 187.379$	$.2044002$

3d. Sought, the time in which L. 15.875 will amount to L. 31.9436 at the rate of 6 per cent.

$$\frac{L.s - L.p}{L.R} = \frac{.3036708}{.0253059} = 12 \text{ years.}$$

4th. At what rate of intereſt per cent. will L. 480 amount to L. 643.246, in the ſpace of 6 years.

$\frac{L.s - L.p}{t} = \frac{.1271358}{6} = .0211893 = L.\ 1.05. = 5$ per cent.

5th.

5th. Required the amount of one farthing lent out at 5 per cent. compound intereſt for the ſpace of 1000 years? Here,

L. $1.05 \times 1000 =$ 21.1893
$+$ L. $.00104166 =$ -3.0177287

L. $pR^t =$ $18.2070287 =$ L. 1610752, &c. to 19 places of figures.

If p the principal conſiſts of pounds, ſhillings, and pence, or t the time, of years, months, and days, theſe parts of an integer may be thrown into

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Of Logarithms.

Logarithms are a moſt valuable invention for ſolving queſtions in trigonometry, or where the higher powers of numbers, as here, are concerned; and for which the learned world are indebted to John Napier, Baron of Merchieſton, in Scotland, about the year 1614.

1ſt. The index of the logarithm of any number is always one leſs than the number of integers it contains; of conſequence the index of the logarithm of any one of the 9 digits is 0; and the index of the logarithm of a decimal fraction is negative, though ſtill equal to the diſtance of its firſt ſignificant figure from the units place, being -1, -2, -3, &c. or the complement thereof to 10, viz. 9, 8, 7, &c.

2d. The logarithm of any number to 7 places of figures may be found by Sherwin's Tables, namely of 5 figures by the Tables, and of two by the column *pts*, nearly exact, and *vice verſa*. Thus,

L. 6484.300 = 3.8118631
L. .080 = 53
L. .006 = 4

L. 6484.386 = 3.8118688

3d. The multiplication or diviſion of natural numbers is perfotmed by the addition or ſubtraction of their logarithms, or, in place of ſubtraction, the addition of the complement of a logarithm may be uſed, providing you caſt 10 out of the index of the ſum. Thus,

L. 20 = 1.3010300
L. 21 = 1.3222193
Co. *L.* 220 = 7.6575773

L. $\frac{20 \times 21}{220} = 0.2808266 = 1.9$

to a decimal fraction, and the anſwer found as above.

Ex. Sought the amount of L. 310: 12: 6, for 5 years and 91 days, at 4 per cent. compound intereſt. Here,

pr^t = L. 1.04 × 5.25 + L. 310.625 = L. 381, 12 s. 11 d. = Amount.

42. The ſeveral particulars reſpecting compound intereſt may more readily be found by the help of the three following Tables, wherein are calculated the amount and preſent worth of L. 1 for any given time, and at ſeveral rates of compound intereſt.

4th. A given power of any number may be found by multiplying the logarithm of the number by the index of the power, the product will be the logarithm of the power ſought. Thus,

$L.\ \overline{32}^3 = 1.50515 \times 3 = 4.5154500 = 32768$, the cube of 32.

In a decimal fraction, the firſt ſignificant figure of the power muſt be put ſo many places below the place of units, as the index of its logarithm wants of 10 × index of the power. Thus,

$L.\ \overline{.25}^4 = 9.39794 \times 4 = 37.5917600 = .00390625$, ſeeing index × 10 = 40.

5th. The root of any power may be extracted by dividing the logarithm of the number by the index of the power, the quotient gives the logarithm of the root ſought. Thus,

$L.\ \sqrt{2304} = \frac{3.3624825}{2} = 1.6812412 = 48$, the root ſought.

In a decimal fraction, prefix to the index of its logarithm a figure leſs by one than the index of the power, and divide as above. Thus,

$L.\ \sqrt[3]{.75} = \frac{29.8750613}{3} = 9.9583533 = .90856$, the root ſought.

6th. Any number of mean proportionals, between any two given numbers, may be found thus: Let a = 1ſt, z = laſt number, n = number of proportions; then $\frac{L.z - L.a}{n+1} = L.r$, which added to $L.a$, n times, gives the proportionals.

TABLE

TABLE I. Shewing the Amount of L. 1 for Days.

Days	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.
1	1.0000809	1.0000942	1.0001074	1.0001206	1.0001336	1.0001596
2	1.0001619	1.0001880	1.0002149	1.0002412	1.0002673	1.0003193
3	1.0002429	1.0002827	1.0003224	1.0003618	1.0004011	1.0004790
4	1.0003240	1.0003770	1.0004299	1.0004824	1.0005348	1.0006387
5	1.0004050	1.0004713	1.0005374	1.0006031	1.0006685	1.0007985
6	1.0004860	1.0005656	1.0006449	1.0007241	1.0008022	1.0009582
7	1.0005671	1.0006600	1.0007524	1.0008447	1.0009360	1.0011180
8	1.0006482	1.0007544	1.0008600	1.0009654	1.0010698	1.0012778
9	1.0007293	1.0008488	1.0009677	1.0010861	1.0012035	1.0014375
10	1.0008104	1.0009433	1.0010754	1.0012068	1.0013372	1.0015973
20	1.0016211	1.0018866	1.0021510	1.0024137	1.0026744	1.0031948
30	1.0024320	1.0028300	1.0032267	1.0036206	1.0040117	1.0047923
40	1.0032430	1.0037733	1.0043024	1.0048276	1.0053489	1.0063898
50	1.0040541	1.0047167	1.0053781	1.0060346	1.0066862	1.0079874
60	1.0048651	1.0056601	1.0064538	1.0072416	1.0080236	1.0095849
70	1.0056762	1.0066036	1.0075295	1.0084486	1.0093609	1.0111825
80	1.0064873	1.0075471	1.0086052	1.0096556	1.0106983	1.0127801
90	1.0072984	1.0084906	1.0096809	1.0108626	1.0120357	1.0143777
100	1.0081096	1.0094342	1.0107567	1.0120697	1.0133731	1.0159753
120	1.0097320	1.0113214	1.0129083	1.0144839	1.0160480	1.0191706
140	1.0113545	1.0132087	1.0150600	1.0168982	1.0187229	1.0223660
160	1.0129771	1.0150961	1.0172118	1.0193126	1.0213979	1.0255615
180	1.0145998	1.0169836	1.0193637	1.0217271	1.0240730	1.0287571
200	1.0162226	1.0188712	1.0215157	1.0241417	1.0267483	1.0319528
220	1.0178455	1.0207589	1.0236678	1.0265564	1.0294237	1.0351486
240	1.0194685	1.0226467	1.0258200	1.0289712	1.0320992	1.0383445
260	1.0210916	1.0245346	1.0279723	1.0313861	1.0347749	1.0415405
280	1.0227148	1.0264226	1.0301247	1.0338011	1.0374507	1.0447366
300	1.0243381	1.0283107	1.0322772	1.0362162	1.0401267	1.0479328
320	1.0259615	1.0301989	1.0344298	1.0386314	1.0428028	1.0511291
340	1.0275850	1.0320872	1.0365825	1.0410467	1.0454790	1.0543255
360	1.0292086	1.0339756	1.0387353	1.0434621	1.0481554	1.0575220
365	1.03	1.035	1.04	1.045	1.05	1.06

TABLE II. Shewing the Amount of L. 1 for Years.

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.
1	1.03	1.035	1.04	1.045	1.05	1.06
2	1.0609	1.071225	1.0816	1.092025	1.1025	1.1236
3	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016
4	1.125509	1.147523	1.169858	1.192518	1.215506	1.262476
5	1.159274	1.187686	1.216653	1.246182	1.276281	1.338225
6	1.194052	1.22925	1.26532	1.30226	1.34009	1.41852
7	1.229874	1.27228	1.31592	1.36086	1.40710	1.50363
8	1.266770	1.31681	1.36857	1.42210	1.47745	1.59385
9	1.304773	1.36289	1.42321	1.48609	1.55133	1.68948
10	1.343916	1.41059	1.48024	1.55296	1.62889	1.79084
11	1.38423	1.45997	1.53945	1.62285	1.71034	1.89830
12	1.42576	1.51107	1.60103	1.69588	1.79585	2.01220
13	1.46853	1.56395	1.66507	1.77219	1.88565	2.13293
14	1.51259	1.61869	1.73167	1.85194	1.97993	2.26090
15	1.55796	1.67535	1.80094	1.93528	2.07893	2.39656
16	1.60471	1.73398	1.87298	2.02237	2.18287	2.54025
17	1.65284	1.79467	1.94790	2.11337	2.29202	2.69277
18	1.70243	1.85749	2.02581	2.20848	2.40662	2.85434
19	1.75350	1.92250	2.10685	2.30786	2.52695	3.02560
20	1.80611	1.98978	2.19112	2.41171	2.65329	3.20713
21	1.86029	2.05943	2.27877	2.52024	2.78596	3.39956
22	1.91610	2.13151	2.36992	2.63365	2.92526	3.60353
23	1.97358	2.20611	2.46471	2.75216	3.07152	3.81975
24	2.03279	2.28333	2.56330	2.87601	3.22510	4.04893
25	2.09378	2.36324	2.66583	3.00543	3.38635	4.29187
26	2.15659	2.44595	2.77247	3.14068	3.55567	4.54938
27	2.22129	2.531567	2.88337	3.28201	3.73345	4.82234
28	2.28792	2.62017	2.99870	3.42969	3.92013	5.11168
29	2.35656	2.711878	3.11865	3.58403	4.11613	5.41838
30	2.42726	2.80679	3.24339	3.74531	4.32194	5.74349
31	2.50008	2.905031	3.37313	3.91385	4.53804	6.08810
32	2.57508	3.00671	3.50806	4.08998	4.76494	6.45338
33	2.65233	3.11194	3.64838	4.27403	5.00319	6.84059
34	2.73190	3.22086	3.79431	4.46636	5.25335	7.25102
35	2.81386	3.33359	3.94609	4.66734	5.51601	7.68608
36	2.89828	3.45026	4.10393	4.87738	5.79181	8.14725
37	2.98522	3.57102	4.26809	5.09686	6.08141	8.63608
38	3.07478	3.69601	4.43881	5.32622	6.38547	9.15425
39	3.16703	3.82537	4.61636	5.56589	6.70475	9.70351
40	3.26204	3.95926	4.80102	5.81636	7.03998	10.28571
41	3.35989	4.09783	4.99306	6.07810	7.39198	10.90286
42	3.46069	4.24126	5.19278	6.35161	7.76158	11.55703
43	3.56451	4.38970	5.40050	6.63744	8.14966	12.25045
44	3.67145	4.54334	5.61651	6.93612	8.55715	12.98548
45	3.78159	4.70236	5.84117	7.24825	8.98500	13.76461
46	3.89504	4.86694	6.07482	7.57442	9.43426	14.59048
47	4.01189	5.03728	6.31781	7.91527	9.90597	15.46591
48	4.13225	5.21356	6.57053	8.27145	10.40127	16.39387
49	4.25622	5.39606	6.83335	8.64367	10.92133	17.37750
50	4.38390	5.58494	7.10668	9.03263	11.46740	18.42015

Table I. and II. ſhew the amount of L. 1 for days or years, and are conſtructed by Equation 1ſt, 41. whereby the tabular number $n = R^t$. Or thus, Table I. is calculated by taking 364 mean geometrical proportionals, betwixt one, and the amount of L. 1. for one year, according to the ſeveral rates of intereſt, or, which is the ſame thing, by dividing the logarithm of R by 365, the natural number anſwering to the quotient will be the amount of L. 1 for one day; and this amount multiplied into itſelf continually will give the amount for any number of days. If a given number of days be not in the Table, divide it into two ſuch numbers as are to be found there, multiply their tabular numbers into one another, and their product will be the amount of L. 1 for the given time. In like manner, Table II. is conſtructed by multiplying the amount of L. 1 for one year, at a given rate, by itſelf continually, the products of which ſhew the amount of L. 1 for their correſponding times. And if the time conſiſts of years and days, their tabular numbers muſt be multiplied into one another to form the amount of their ſum. From the nature of the Tables, $1 : n :: p : np = s$, the amount of any principal p, for the time and at the rate correſponding to n. Hence the following

TABLE.

Given.		
$p, t, r,$	$s = np.$	r being given, t is found
$s, t, r,$	$p = \frac{s}{n}.$	oppoſite to n; and t being
$t, r, p,$	$n = \frac{s}{p}.$	given, r is found above n
$s, t, p,$		in the tables.

Ex. 1ſt,

Ex. 1ſt. What will L. 500 amount to in 15 years, at five per cent. compound intereſt? Here,

$$np = 2.07893 \times 500 = \text{L. } 1039.465.$$

Ex. 2d. What will be the amount of L. 419.2965 in the ſpace of 20 years and 9 months, at $4\frac{1}{2}$ per cent. Here,

$$n \times n = 2.49265 = 0.3966608$$
$$p = 419.2965 = 2.6224885$$

$$\text{L. } nnp = 3.0191493 = \text{L. } 1045.08.$$

Ex. 3d. What principal will amount to L. 1039.464 in 15 years, at 5 per cent? Here,

$$\frac{s}{n} = \frac{1039.464}{2.07893} = \text{L. } 500.$$

Ex. 4th. At what rate of intereſt will L. 25 amount to L. 44.896 in the ſpace 12 years? Here,

$\frac{s}{p} = \frac{44.896}{25} = 1.79585$, found under 5 per cent.

Ex. 5th. In what time will L. 300 amount to L. 500, at the rate of 5 per cent? Here,

$\frac{s}{p} = \frac{500}{300} = 1.66666$, correſponding to 10 years.

and $\frac{1.66666}{1.62889} = 1.0232$, correſponding to 172 days.

TABLE

TABLE III. Shewing the present Worth of L. 1 for Years.

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.
1	.970874	.966183	.961538	.956938	.952381	.943396
2	.942596	.933510	.924556	.915729	.907029	.889996
3	.915141	.901942	.888996	.876296	.863837	.839619
4	.888487	.871442	.854804	.838561	.822702	.792093
5	.862608	.841973	.821927	.802451	.783526	.747258
6	.837484	.813500	.790384	.767895	.746215	.704960
7	.813091	.785991	.759918	.734828	.710681	.665057
8	.789409	.759411	.730690	.703185	.678939	.627412
9	.766416	.733731	.702586	.672904	.644609	.591898
10	.744093	.708918	.675564	.643927	.613913	.558394
11	.722421	.684945	.649581	.616198	.584679	.526787
12	.701379	.661783	.624597	.589663	.556837	.496969
13	.680950	.639404	.600574	.564271	.530321	.468839
14	.661117	.617781	.577475	.539973	.505068	.442301
15	.641860	.596190	.555264	.516720	.481017	.417265
16	.623167	.576706	.533908	.494469	.458111	.393646
17	.605016	.557203	.513373	.473176	.436296	.371364
18	.587394	.538361	.493628	.452800	.415520	.350344
19	.570286	.520155	.474642	.433302	.395734	.330513
20	.553675	.502566	.456387	.413643	.376889	.311804
21	.537549	.485571	.438833	.396787	.358942	.294155
22	.521892	.469150	.421955	.379701	.341849	.277505
23	.506691	.453285	.405726	.363350	.325571	.261797
24	.491933	.437957	.390121	.347703	.310068	.246978
25	.477605	.423147	.375117	.332730	.295303	.232998
26	.463694	.408837	.360689	.318402	.281240	.219810
27	.450189	.395012	.346816	.304691	.267848	.207368
28	.437076	.381654	.333477	.291571	.255093	.195630
29	.424346	.368748	.320651	.279015	.242946	.184556
30	.411986	.356278	.308318	.267000	.231377	.174110
31	.399987	.344230	.296460	.255502	.220359	.164255
32	.388337	.332589	.285058	.244500	.209866	.154957
33	.377026	.321342	.274094	.233971	.199872	.146186
34	.366045	.310476	.263552	.223896	.190355	.137911
35	.355383	.299977	.253415	.214254	.181290	.130105
36	.345032	.289833	.243668	.205028	.172657	.122741
37	.334983	.280031	.234297	.196199	.164435	.115793
38	.325226	.270562	.225285	.187750	.156605	.109238
39	.315753	.261412	.216620	.179665	.149148	.103055
40	.306556	.252572	.208289	.171928	.142045	.097222
41	.297628	.244031	.200278	.164525	.135281	.091720
42	.288959	.235779	.192575	.157440	.128840	.086527
43	.280543	.227806	.185168	.150660	.122704	.081629
44	.272372	.220102	.178046	.144173	.116861	.077009
45	.264438	.212660	.171198	.137964	.111296	.072650
46	.256736	.205468	.164614	.132023	.105996	.068538
47	.249258	.198520	.158282	.126338	.100949	.064658
48	.241998	.191806	.152195	.120897	.096142	.060998
49	.234950	.185320	.146341	.115691	.091564	.057545
50	.228107	.179053	.140713	.110709	.087204	.054288

Table III ſhews the preſent value of L. 1 for years, and is conſtructed by Equation 2d, 41. whereby the tabular number $p = \frac{1}{R^t}$. Or it is the reciprocal of Table II. whereby n of Table III. $= \frac{1}{s}$ of Table II. From the nature of the Table $1 : n :: s : ns = p$, the principal or preſent worth of s, for the time and at the rate correſponding to n. Hence the following

TABLE.

Given.	
$s, t, r,$	$p = ns.$
$p, t, r,$	$s = \frac{p}{n}.$
$s, r, p,$ / $s, t, p,$	$n = \frac{p}{s}.$

Ex. 1. Required the preſent worth of L. 250, due 10 years hence, diſcounting at the rate of $4\frac{1}{2}$ per cent. compound intereſt.

$ns = .643927 \times 250 =$ L. 160.98175 preſent worth.

Ex. 2. In what time will L. 350 amount to L. 571.458, at 4 per cent compound intereſt?

Here $\frac{p}{s} = \frac{350}{571.458} = .61247$ correſponding to 12 years and an half.

Theſe examples may, with equal facility, be determined by the equations annexed to Table II.

Re-

Remarks upon Compound Intereſt.

43. 1ſt. A ſum of money put out at compound intereſt ought, by the doctrine of fluctions, to accumulate every inſtant, at the rate of the inſtant; ſo as to reſemble an organized body in its gradual and conſtant increaſe. Upon this ſuppoſition let $\dot{r}$ be the fluction of the rate, or the rate of the inſtant,

Then $1+\dot{r}$ = amount of L. 1 in one inſtant multiplied by $1+\dot{r}$

produces $1+2\dot{r}+\ddot{r}r$ = the fluction of that amount at the end of the firſt year; whoſe fluent, viz. $1+2r$ = amount of L. 1 at the end of the year. Hence the intereſt of money, when the times and rates are the ſame, will, when it accumulates every inſtant, be double to what it would be when it accumulates only once in the year. But as this mode might elude the graſp of calculation, and for the benefit of all concerned, the accumulation of money hath been extended to the twelvemonth, and only takes place once in the year.

2d. Although there be no particular theorem for diſcovering the compound intereſt of a ſum of money diſtinct from the amount, yet it may eaſily be found, being the difference betwixt the principal and the amount. In like manner, was it lawful to diſcount money on the principles of compound intereſt, the diſcount might be found by ſubtracting the preſent worth from the ſum of the bill.

3d. If the time be ſought in which a ſum of money will double itſelf at the ſeveral rates of compound intereſt; in this caſe, as the amount is equal to twice the principal, $pR^t = 2p$, which gives $R^t = 2$, and $t = \frac{L.2}{L.R}$. Hence,

$s = 2p$, when $t = \frac{L.2}{L.R} =$

23.45—	3
20.149—	$3\frac{1}{2}$
17.67+	4
15.8—years at	$4\frac{1}{2}$ per cent.
14.206+	5
11.98+	6
34.886+	2

4th. The amount of a ſum of money, *ceteris paribus*, is always greater, and its preſent value leſs, upon the principles of compound than of ſimple intereſt, in proportion of $\overline{1+r}^t$, to $1+tr$; and of $\frac{1}{1+r^t}$, to $\frac{1}{1+tr}$. And as this difference is very conſiderable, ſeeing $\frac{L.2}{L.R}$ years intereſt of the one is equal to $\frac{1}{r}$ years intereſt of the other, the charging compound intereſt upon any bill hath been expreſsly prohibited by acts of parliament, at the riſque of forfeiting both principal and intereſt. Notwithſtanding of this, compound intereſt enters deeply into the tranſactions of men in their affairs of property. Thus, not only the ſale of annuities, and the purchaſe of perpetuities are calculated upon the principles of compound intereſt; but alſo where caſh-accounts are balanced, or the intereſt of money and the rent of an eſtate received yearly, there you have the advantage of compound intereſt, ſeeing you have the uſe, and can diſpoſe of that intereſt.

CHAP.

CHAP. VI.

Of ANNUITIES *computed at* COMPOUND INTEREST.

1*ſt*, *Of the Amount of an Annuity in Arrears.*

44. IN computing annuities at compound intereſt, let a repreſent an annuity, penſion, or rent, &c. in arrears; then, ſince the firſt year's annuity bears no intereſt, the laſt year's annuity will be repreſented by aR^{t-1}; and deriving a ſeries from the quantity a, in the ratio of 1 : R, we ſhall have $a + aR + aR^2 + aR^3 + aR^4 \ldots + aR^{t-1} = s$ which gives $s = a \times \frac{R^t - 1}{R-1}$. Hence the following

TABLE.

Given.		
a, t, R,	$s = a \times \frac{R^t-1}{R-1} = a \times \frac{R^t-1}{r} = \frac{a}{r}R^t - \frac{a}{r}$	Equat. 1.
s, t, R,	$a = rs \times \frac{1}{R^t-1}$ . . .	2.
s, t, a,	$\frac{s}{a}R - R^t = \frac{s}{a} - 1$ . . .	3.
s, R, a,	$t = \frac{L.\overline{\frac{rs}{a}+1}}{L.R}$ . . .	4.

Sought s in Equation 1ſt.

38. $s - a = \overline{s - aR^{t-1}} \times R$.
$s - a = Rs - aR^t$.
$Rs - s = aR^t - a$.
$\overline{R-1} \times s = a \times \overline{R^t - 1}$.
but $R = 1 + r$, &c. Eq. 2.

Sought R in Equation 3.

38. $\overline{R^t - 1} \times a = Rs - s$.
$R^t - 1 = \frac{s}{a}R - \frac{s}{a}$.

Sought t in Equation 4th.

$\overline{R^t - 1} \times a = rs$.
$R^t - 1 = \frac{rs}{a}$ & $R^t = \frac{rs}{a} + 1$.

Ex. 1ſt. It is required to find the amount of an annuity L. 5.25 per annum, at the end of 30 years, reckoning intereſt at 4 per cent? Here $a = 5.25$, $t = 30$, $R = 1.04$ and $r = .04$.

$a \times \frac{R^t - 1}{r} = 5.25 \times \frac{2.243}{.04} =$ $\quad L.\ R = .0170333$, $t = 30$

$= $ L. 297.42 the amount. $\quad L.\ R^t = .5109990 = 3.2433$

$R^t - 1 = 2.2433$

Ex. 2d. Sought, the yearly annuity which in the ſpace of 15 years will amount to L. 1000, at the rate of $4\frac{1}{2}$ per cent. Here, by Equation 2d,

$rs \times \frac{1}{R^t - 1} = 45 \times \frac{1}{.93528} =$ $\quad L.\ R = .0191163$, $t = 15$

$45 \times 1.0692 =$ L. 48.114 $\quad .2867445 = 1.93528$, $10 - 1.$

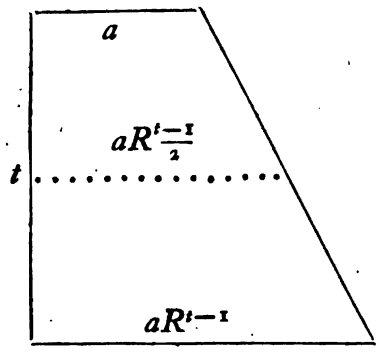
$L.\ .93528 = 9.9709416$ $\quad .93528 = R^t - 1$

$.0290584 = 1.0692 = \frac{1}{R^t - 1}$

Ex. 3d. In what time will an yearly annuity of L. 48.114 amount to L. 1000 at the rate of $4\frac{1}{2}$ per cent. compound intereſt?

Equation 4th, $\frac{L.\frac{rs}{a} + 1}{L.\ R} = \frac{L.\ 1.93528}{L.\ 1.045} = \frac{.2867445}{.0191163} = 15$ years.

45. Given, the annuity, time, and amount; ſought, the rate of compound intereſt. In this caſe, as R, in Equation 3d, is involved to a high power, and the Equation is of a mixed nature, the



the value of R may be found in this manner. The amount of an annuity may be conceived to be nearly equal to the area of a geometrical figure, having two of its ſides parallel and two of its angles right; whoſe leaſt breadth $= a$, its greateſt $= aR^{t-1}$, and its height $= t$. Hence $s = atR^{\frac{t-1}{2}}$, nearly; which gives $\frac{s}{at} = R^{\frac{t-1}{2}}$, and $\frac{t-1}{2}\sqrt{\frac{s}{at}} = R = 1 + r$, nearly.

Ex. 4th. An annuity of L. 34.4 in arrears 12.5 years, amounts to L. 614.4328; it is required to find the rate of intereſt allowed. Here

$L.\ \frac{t-1}{2}\sqrt{\frac{s}{at}} = .0269575 = L.s = 2.7884743$
$1.064 = R = 1 + r = L.\ at = 2.6334685$

6 per cent. nearly. $\frac{t-1}{2} = 5.75)0.1550058(.0269575$

If greater exactneſs be deſired, let $\frac{6}{t+1} = b$, then $\sqrt{bb + 2rb} - b = r$ corrected. Or thus *,

Aſſume

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* *The Rule of* DOUBLE POSITION.

Let P and p, the two poſitions, and x the number ſought, be equally increaſed or diminiſhed, by any operation of arithmetic, producing thereby M, m and $\mathfrak{Q}$, their reſpective reſults; then it is obvious, that

$$M \text{ on } m : P \text{ on } p :: \begin{cases} M \text{ on } \mathfrak{Q} : P \text{ on } x. \\ m \text{ on } \mathfrak{Q} : p \text{ on } x. \end{cases}$$

Let

Aſſume two poſitions, find their reſpective reſults, and work by the following proportion.

As the difference of the reſults of the two poſitions : is to the difference of the poſitions : : ſo is the difference of the reſults of the firſt or of the ſecond poſition, and of the number ſought : to the difference of the firſt or of the ſecond poſition and number ſought.

Ex. 5th. An annuity of L. 20 per annum, 7 years in arrears, is offered to be ſold for L. 180.5; what rate of intereſt is allowed to the purchaſer in this bargain? Here R is ſought.

1ſt poſition, $R = 1.08$ ... reſult 178.456... error $-$ 2.044
2d poſition, $R = 1.085$... reſult 181.210... error $+$ 0.710

By the queſtion reſult 180.500, diff. of reſ. $= 2.754$

hence

Let $M \sim \mathfrak{Q} = R$, and $m \sim \mathfrak{Q} = r$ be the errors of the two reſults M and m, which are either $+$ or $-$; then

$$M \sim m = \left\{ \begin{matrix} R \sim r \\ R + r \end{matrix} \right\} \text{ when the errors are } \left\{ \begin{matrix} \text{alike.} \\ \text{unlike.} \end{matrix} \right.$$

Ex. Suppoſe $2x + \frac{x}{2} + 2 + \frac{1}{2} = 20$; ſought x.

1ſt poſition, $P = 9$... $M = 25$ ... $R = +5.0$
2d poſition, $p = 6$... $m = 17.5$... $r = -2.5$

ſought x ... $\mathfrak{Q} = 20$... $M \sim m = 7.5$

hence, $7.5 : 3 : : 5 : 2 = P \sim x$, which gives $x = 7$, when the errors $+$ and $-$ are equal, $\frac{P+p}{2} = x$.

When the higher powers of the unknown quantity, x, are concerned in the queſtion, this method can only give an anſwer near the truth; ſeeing $P^2 : p^2$ is in a duplicate, and $P^3 : p^3$, in a triplicate ratio of $P : p$.

The above proportion is alſo the foundation of a ſecond well-known rule in double poſition.

RULE

hence 2.754 : .005 : : 2.044 : .0037, to be added to the firſt poſition, making $R = 1.0837$; which ſhews that the rate of intereſt is L. 8.37 per cent.

46. When an annuity is paid, which is uſually the caſe, every half-year, quarterly, or monthly, its amount may be found by the Equation, $s = a \times \frac{R^t - 1}{r}$; Here a is the half-year's, or quarterly payment, R^t is the ſame as if the annuity had

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RULE.

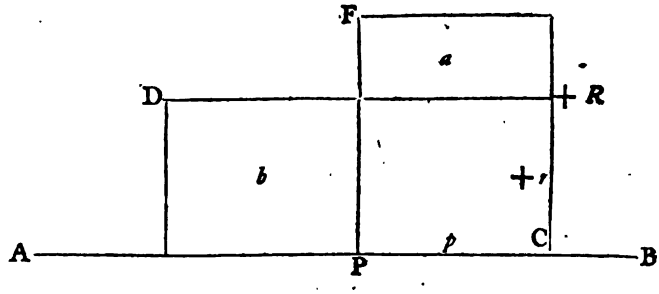
Divide the difference of the products of the two poſitions, multiplied into their alternate errors, by the difference of their reſults, that is, by the difference of the errors, when they are alike; or divide the ſum of the products by the difference of their reſults, that is, by the ſum of the errors, when they are unlike; the quotient in either caſe will give the anſwer. Thus,

$\frac{rP \mp Rp}{M \mp m = R \mp r} = x$ { when the errrors are alike, that is both +, or both —.

$\frac{rP + Rp}{M \mp m = R + r} = x$ { when the errors are unlike, that is one + and the other —.

Ex. When the errors are both +, or both —.

1ſt poſition, $= P$, its error $+ R$.
2d poſition, $= p$, its error $= r$.
x is ſought.



Let the right line AB be conceived to terminate the true reſult by the queſtion; then the errors + will fall above, and — below that line; the rectangles CD + CF $= rP + Rp$, are the products of the two

had been paid yearly, ſeeing R^t for years is equal to R^{4t} for quarters; but the diviſor muſt always correſpond to the payments, and may be diſcovered by the following

TABLE. Shewing the Amount of L. 1 for the even Parts of a Year.

M.	W.	D.	3 per cent.	3½ pr cent.	4 per cent.	4½ pr cent.	5 per cent.	6 per cent.
		1	1.0000809	1.0000942	1.0001074	1.0001206	1.0001336	1.0001596
	1	7	1.0005670	1.000660	1.0007524	1.0008445	1.0009361	1.001118
	2	14	1.001134	1.001320	1.001505	1.001690	1.001873	1.002238
	3	21	1.001472	1.001981	1.002258	1.002535	1.002811	1.003360
	4	28	1.002270	1.002642	1.003012	1.003382	1.003750	1.004480
1			1.002466	1.002871	1.003274	1.003675	1.004074	1.004867
3	13		1.007417	1.008637	1.009853	1.011065	1.012272	1.014674
6	26		1.014889	1.017350	1.019840	1.022253	1.024694	1.029563
9	39		1.022417	1.026137	1.029852	1.033564	1.037270	1.044671
12	52	365	1.03	1.035	1.04	1.045	1.05	1.06

In this Table, which is the ſame in conſtruction with Table I. whilſt the tabular numbers ſhew R the amount, the decimals point out r the intereſt of L. 1 for their correſponding days, months, or quarters, at the ſeveral rates of intereſt.

Ex. 1ſt, A ſervant, who was engaged to ſerve at 5 s. per week, after 7 years received his wages, with intereſt upon the weekly payments, at the

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two poſitions multipled into their alternate errors, and $a \circledcirc b$ is the difference of theſe products. But $a = p \times R \circledcirc r$, which gives $\frac{a}{R \circledcirc r} = p$ and $b = P \circledcirc p \times r$. Again, by the above proportion, $M \circledcirc m = R \circledcirc r : P \circledcirc p :: r : p \circledcirc x$; which gives $\frac{b}{R \circledcirc r} = p \circledcirc x$. Hence $\frac{a \circledcirc b}{R \circledcirc r} = (p \circledcirc \overline{p \circledcirc x} =) x$; that is, the difference of the products, when the errors are alike, divided by the difference of the errors, will quot the anſwer. In like manner it might be ſhewn, when the errors are unlike, that $\frac{a+b}{R+r} = (\frac{P+p}{2} - \frac{P+p}{2} \circledcirc x =) x$.

rate

rate of 5 per cent. per annum; ſought, the amount of his wages, and intereſt thereon. Here $a = .25$, $R^t = 1.4071$ and $r = .000936$. Hence,

$$a \times \frac{R^t - 1}{r} = .25 \times \frac{.4071}{.000936} = \text{L. } 108.734.$$

Ex. 2d. A ſailor, being engaged for a voyage of 6 years, at the rate of L. 1, 10 s. per month, at his return to port, received his wages, with intereſt upon his monthly payments, at the rate of 4 per cent: per annum: Sought, the amount of the whole.

Here $a = 1.5$, $R = \overline{1.04}^{6} = 1.26532$, and $r = .003274$.
Hence $a \times \frac{R^t - 1}{r} = 1.5 \times \frac{.26532}{.003274} = \text{L. } 121.557$.

Ex. 3d. What will an annuity of L. 17.5, payable every quarter of a year, amount to in 5 years, at five per cent per annum? Here,

$a = 17.5$, $R^t = \overline{1.05}^{5} = 1.27628$, and $r = .012272$.
Hence $17.5 \times \frac{.27628}{.012272} = \text{L. } 393.977$.

In like mannner, *mutatis mutandis*, may a, t, and R be found from the other equations of 44th, when the payments are made every half year, quarterly, &c.

2d. *Of the preſent Worth of an Annuity computed at Compound Intereſt.*

47. Let p repreſent the preſent worth of an annuity a; then, ſince $1 : \frac{1}{R}$ the preſent worth of L. 1, as $a : \frac{a}{R}$, this at the end of one year will be the preſent worth of a; and deriving a ſeries from

the quantity $\frac{a}{R}$ in the ratio of $1 : \frac{1}{R}$, we ſhall have $\frac{a}{R} + \frac{a}{R^2} + \frac{a}{R^3} + \frac{a}{R^4} \ldots + \frac{a}{R^t} = p$ the preſent worth of the annuity a, which gives $p = a \times \frac{1 - \frac{1}{R^t}}{R - 1} = a \times \frac{1 - \frac{1}{R^t}}{r}$. Or thus, the ſeller, when the annuity ceaſes, has pR^t; the purchaſer, at the end of the time, has $a \times \frac{R^t - 1}{r}$, but by ſuppoſition $pR^t = a \times \frac{R^t - 1}{r}$, which gives $p = a \times \frac{R^t - 1}{r R^t} = a \times \frac{1 - \frac{1}{R^t}}{r}$. Hence the following

TABLE.

Given.		
$a, t, R,$	$p = a \times \frac{1 - \frac{1}{R^t}}{r} = \frac{a}{r} - \frac{a}{rR^t}$.	Equation 1.
$p, t, R,$	$a = rp \times \frac{1}{1 - \frac{1}{R^t}}$	2.
$p, t, a,$	$\overline{1 + \frac{a}{p}} \times R^t - R^{t+1} = \frac{a}{p}$.	3.
$p, r, a,$	$t = \frac{L.\ a - L.\ \overline{a - rp}}{L.\ R}$	4.

Sought p in Equation 1ſt.

$p - \frac{a}{R^t} = \overline{p - \frac{a}{R}} \times R$.

$p - \frac{a}{R^t} = Rp - a$.

$Rp - p = a - \frac{a}{R^t}$.

$\overline{R - 1} \times p = a \times \overline{1 - \frac{1}{R^t}}$. Eq. 2.

Sought R in Equation 3d.

$p - \frac{a}{R^t} = Rp - a$.

$Rp + \frac{a}{R^t} = a + p$.

$p \times R^{t+1} + a = aR^t + pR^t$.

$R^{t+1} + \frac{a}{p} = \frac{a}{p} R^t + R^t$.

Sought t in Equation 4th.

$rp = a - a\frac{1}{R^t}$.

$a\frac{1}{R^t} = a - rp$.

$a = \overline{a - rp} \times R^t$, and $t =$ &c.

Ex. 1ſt.

Ex. 1ſt. What is the preſent worth of an yearly annuity of L. 20 to continue 20.75 years, reckoning intereſt at $4\frac{1}{2}$ per cent?

$a \times \frac{1-\frac{1}{R^t}}{r} = 20 \times \frac{.598822}{.045} = L.\ R^t = .3966632$ 1.00

$L.\ \frac{1}{R^t} = 9.6033368 = 0.401178$

$1-\frac{1}{R^t} = .598822$

L. 266.14 the preſent value.

If the years purchaſe which an annuity is worth, be ſought; they are manifeſtly equal to the number of times the annuity is contained in its preſent value. So that n, the number of years purchaſe, $= \frac{p}{a} = \frac{1-\frac{1}{R}}{r} = n$ of Table V. Thus in the laſt example, $\frac{266.14}{20} = \frac{.59882}{.045} = 13.307 =$ years purchaſe.

Ex. 2d. A gentleman is willing to ſpend L. 1000 upon the purchaſe of an annuity, to continue 15 years, being allowed diſcount at the rate of $4\frac{1}{2}$ per cent.; ſought, the annuity.

$rp \times \frac{1}{1-\frac{1}{R^t}} = 45 \times 2.096 = L.\ R^t = .2867445$ 1.00

$L.\ \frac{1}{R^t} = 9.7132555 = 0.51672$

$= L.\ 93.114$ annuity. $1-\frac{1}{R^t} = .48328 = 9.6841988$

$2.0962 = .3158012$

Ex. 3d. Sought the duration of an yearly annuity of L. 80, whoſe preſent value is L. 800; diſcounting intereſt at 5 per cent.:

Here $\frac{L.\,a - L.\,a - rp}{L.\,R} = \frac{.1010300}{.0211893} = 14$ years 75 days.

48. Given, the annuity, its preſent worth and time; ſought, the rate of intereſt per cent.

Caſe 1ſt, If the number of years be great, as 40 or upwards, and if the rate of intereſt be high; in this caſe, $\frac{1}{R^t}$ will be of ſmall value, and $\frac{a}{p} = r$,

with this rate find the preſent value of the reverſion of the annuity, viz. $\frac{a}{rR^t} = x$; then $\frac{a}{p+x} = r$, the rate ſought.

Ex. 4. Suppoſe a = L. 70, p = L. 1321.3028, and t = 59 years; ſought r, the rate of intereſt.

$\frac{a}{p} = .052978 = r$; $\frac{a}{rR^t} = \frac{70}{111139} =$ L. 62.844 $= x$, the reverſion; and $\frac{a}{p+x} = .05057 = r$ corrected $= 5$ per cent.

Caſe 2. If the number of years is ſmall, as 20 or under, the rate may be found thus:

The preſent worth of an annuity may alſo be conceived to be nearly equal to the area of a geometrical figure, having two of its ſides parallel, and two of its angles right; its greateſt breadth $= \frac{a}{R}$, its leaſt $= \frac{a}{R^t}$, and its height $= t$; hence $p = \frac{at}{R^{\frac{t+1}{2}}}$, which gives $\sqrt[\frac{t+1}{2}]{\frac{at}{p}} = R = 1 + r$, nearly.

Ex. 5. Suppoſe a = L. 20, p = L. 220, and t = 21 years. Sought, the rate of intereſt allowed.

$$\begin{array}{rl} L.\ at, & = 2.6232493 \\ +co.\ L.\ p. & = 7.6575773 \end{array}$$

$\frac{t+1}{2} = 11$) 0.2808266 (.0255297 = 1.0605 = $R = 1 + r = 6$ per cent. If greater accuracy be deſired, let $\frac{6}{t-1} = b$, then $b - \sqrt{1b - 2b} = r$ corrected. Theſe two caſes may alſo be reſolved by the Rule of double poſition.

Ex. 6. Suppoſe a = L. 20, p = L. 100, and t = 7 years; ſought, R.

1ſt

1st position, $R=1.09\ldots$ result $100.659\ldots$ error $+.659$
2d position, $R=1.0925\ldots$ result $99.82\ldots$ error $-.180$

Per question. result 100. diff. of result $=.839$

Hence, $.839 : .0025 :: .180 : .000536$ = difference between the 2d position and R; but seeing, in this question, the greater the interest is, *cæteris paribus*, the less will be the result, this difference must be deducted from 1.0925, making $R=1.091964$; which gives $p=$L. 99.9989.

49. When an annuity is paid every half year, quarterly, or monthly, &c. its present value may be found by the equation $p=a\times\frac{1-\frac{1}{R^t}}{r}$.

Ex. What is the present worth of an annuity of L. 5 paid every quarter, to continue $20\frac{3}{4}$ years, reckoning interest at $4\frac{1}{2}$ per cent.? Here $a=5$, $R^t=\overline{1.045}^{20.75}$ and $r=.011065$; hence $5\times\frac{.598822}{.011065}=$ L. 270.59, the present worth.

By comparing this with Ex. 1st, 47. it appears, that an annuity paid every half year or quarterly is more valuable than when paid yearly. See Chap. VIII. Prob. XX.

In like manner may a, t, and R, be found by the other equations of 47th, when the payments are made every half year, quarterly, or monthly.

3d. *Of the present Worth of an Annuity in reversion.*

50. When an annuity is to commence after a certain number of years, (n) and then to continue for a certain time, (t) its present value is found by

by ſubtracting the preſent worth for n years, from that for $t+n$ years: thus, $a \times \frac{1-\frac{1}{R^{t+n}}}{r} - a \times \frac{1-\frac{1}{R^n}}{r} = p$. Or, divide the preſent value of the annuity at its commencement by R^n, the quotient gives p: Thus, $a \times \frac{1-\frac{1}{R^t}}{r R^n} = a \times \frac{R^t-1}{r R^{t+n}} = p$. Hence the following

TABLE.

Given.		
$a, n, t, R,$	$p = a \times \frac{1-\frac{1}{R^t}}{r R^n} = a \times \frac{1-\frac{1}{R^t}}{R^{n+t}-R^n}$	Equation 1.
$p, n, t, R,$	$a = r R^n p \times \frac{1}{1-\frac{1}{R^t}}$.	2.
$p, n, t, a,$	$\frac{R^{n+t}-R^n}{1-\frac{1}{R^t}} = \frac{a}{p}$.	3.
$p, R, n, a,$	$t = \frac{L.a - L.\overline{a - r R^n p}}{L.R}$.	4.
$p, R, t, a,$	$n = \frac{L.\overline{a - \frac{a}{R^t}} - L.rp}{L.R}$.	5.

Ex. 1. There is an yearly annuity of L. 30, which, commencing after the expiration of a leaſe of 10, is to continue 19 years; it is required to find the preſent value thereof, reckoning intereſt at 5 per cent. Here $a = 30$, $n = 10$, $t = 19$, and $R = 1.05$: Hence $p = 30 \times \frac{.604266}{.0814447} =$ L. 222.58, the preſent value.

Ex. 2. An annuity, which, commencing 12 years hence, is to continue 20 years, being expoſed

posed to sale, sold for L. 250. Reckoning interest at 4 per cent. sought the annuity.

Here $p=250$, $n=12$, $t=20$, $R=1.04$, sought a

L. $rR^n p$ $=1.2044002$

Equ. 2. L. $\frac{1}{1-\frac{1}{R^t}}=$ $.2647102$

$1.4691104=$L. 29.4517, the ann.

Ex. 3. When 12 years of a lease of 21 years were expired, a renewal for the same term was granted for L. 500; 8 years are now expired, and for what sum must a corresponding renewal be made, reckoning 5 per cent. compound interest? Here,

1st, Given $p=500$, $n=9$, $t=12$, $R=1.05$, sought a

Equ. 2. $p\times\frac{rR^t+r^n}{R^t-1}=500\times\frac{.139298}{.79585}=$L. 87.5, the ann.

2. Given $a=87.5$, $n=13$, $t=8$, sought p.

Equ. 1. $a\times\frac{R^t-1}{rR^t+r^n}=87.5\times\frac{.47746}{.139298}=$L. 299.9, the fine.

When a, p, n, and t are given, R may be found by approximation: Thus, let $T=2n+t+1$, then $\frac{T}{2}\sqrt{\frac{at}{p}}=R=1+r$ nearly. If greater exactness be desired, let $\frac{6T+6}{n}=b$, then $b-\sqrt{bb-2rb}$ is exceeding near the value of r.

Ex. 4th, An annuity of L. 40 per annum, being in possession for the term of 21 years, for L. 80 paid down can be prolonged for 10 years more, or to 31 years; what is the rate of interest required? Here $a=40$, $p=80$, $n=21$, and $t=10$, sought R.

$L.\ at = 2.6020600$ $\quad T = 42 + 11 = 53$

$co.\ L.\ p = 8.0969100$

$\frac{T}{2} = 26.5 \big) 0.6989700 \big(0.0263762 = L.\ 1 + r = 1.062616.$

$b = \frac{6T+6}{tt} = 3.24$ $\quad 2r = .125232.$

$\overline{b - 2r} \times b = 3.114768 \times 3.24 = 10.091848 = bb - 2rb.$

$b = 3.24$

$\sqrt{bb - 2rb} = 3.176767$

$.063233 = r$, the rate fought.

Ex. 5. Given $a = $L. 87.5, $p = $L. 500, $n = 9$ years, $R = 1.05$, fought t, the time of the annuity's continuance.

Equ. 4. $\frac{.2542796}{.0211893} = 12$ years.

Ex. 6. Given $a = $L. 20, $p = $L. 145.836, $t = 12$ years, $R = 1.05$, fought n, the number of years prior to the commencement of the annuity.

Equ. 5. $\frac{.0847572}{.0211893} = 4$ years.

Of

Of the Renewing of Leaſes for a certain Number of Years.

A TABLE, ſhewing the Fine for renewing of any Number of Years elapſed in a Leaſe of 21 Years.

Years elapſed.	L.11 : 11s : 8¼ per cent.	5 per cent.	6 per cent.	8 per cent.	10 per cent.	
1	0.1000	0.3590	0.2942	0.1987	0.1352	
2	0.2118	0.7359	0.6060	0.4122	0.2848	
3	0.3364	1.1317	0.9365	0.6489	0.4473	
4	0.4754	1.5471	1.2869	0.8942	0.6272	
5	0.6306	1.9834	1.6583	1.1544	0.8250	
6	0.8035	2.4415	2.0419	1.4563	1.0426	
7	1.0000	2.9226	2.4691	1.7726	1.2820	
8	1.2121	3.4276	2.9114	2.1120	1.5454	
9	1.4526	3.9576	3.3803	2.4807	1.8350	
10	1.7210	4.5148	3.8772	2.8779	2.1437	
11	2.0205	5.0995	4.4040	3.3067	2.5042	
12	2.3446	5.7134	4.9624	3.7699	2.8897	
13	2.7275	6.3580	5.5543	4.2702	3.3038	
14	3.1435	7.0349	6.1817	4.8104	3.7803	
15	3.6077	7.7456	6.7468	5.3939	4.2935	
16	4.1258	8.4917	7.5417	6.0241	4.8579	
17	4.7038	9.2752	8.2990	6.7047	5.4789	
18	5.3488	10.0980	9.0911	7.4397	6.1619	
19	6.0686	10.9618	9.9207	8.2336	6.9132	
20	6.8716	11.8678	10.8107	9.0809	7.7396	
21	7.7682	12.8212	11.7641	10.0168	8.6487	Total value.

In the conſtruction of this table, let n be the number of years in *eſſe*, or yet to run by the leaſe, t the time elapſed ſince its commencement; then from the preſent value of L. 1 annuity for

$n+t$ years, at a given rate, ſubtract that of an annuity for n years; the remainder $=N$, the tabular number expreſſing the fine for renewing the leaſe; the yearly rent being L. 1.

Ex. There is a leaſe of church-lands, for the ſpace of 21 years, at L. 70 per annum, whereof 14 years are expired: It is required to know what fine ought to be paid, to have the ſame renewed, reckoning intereſt at L. 11 : 11 : $8\frac{1}{4}$ per cent.

Let a expreſs the rent, and N the tabular number for 14 years; then

$aN = 70 \times 3.1435 =$ L. 220.045, the fine required.

When the length of the leaſe is different from 21, ſuppoſe 31 years, whereof t years are elapſed: In this caſe, from the preſent value of L. 1 annuity for $n+t$ $(=31)$ years, at a given rate, Tab. V. ſubtract that of an annuity for n years yet to run; the remainder will ſhew the fine for renewing the leaſe, ſuppoſing the rent to be L. 1.

When the fine for the whole leaſe is a given ſum, for inſtance L. 500; ſee Ex. 3d; find a, the annuity which that ſum will purchaſe, at a given rate, during the currency of the leaſe; then a multiplied by N, correſponding to the years elapſed, will produce the fine.

4th, Of the preſent Worth of the Reverſion of an Annuity.

51. The reverſion of an annuity being the ſame with a reverſion in fee-ſimple, which is to commence after a certain time, t, $=$ the duration of the annuity, its preſent value may be found by ſubtracting

tracting the value of the annuity from that of the perpetuity, the remainder gives the reverſion: Or rather thus, Divide the preſent value of the perpetuity, viz. $\frac{a}{r}$ (56.) by R^t, the quotient gives the reverſion; thus $\frac{a}{rR^t}=p$, the reverſion.

Ex. 1*ſt*, Sought the reverſion of an yearly annuity of L. 20, which is to continue 12 years, reckoning intereſt at 5 per cent.

$$\frac{a}{rR^t}=\frac{20}{.0897925}=\text{L } 222.735, \text{ the reverſion.}$$

Ex. 2*d*, Sought, the reverſion of an yearly annuity of L. 40, which is to commence 21 years hence, and to continue 10 years, reckoning intereſt at 6 per cent. Here $a=40$, $n+t=31$, and $R=1.06$.

$$\frac{a}{rR^{n+t}}=\frac{40}{.365286}=\text{L. } 109.5, \text{ the reverſion.}$$

See reverſions more fully treated of in 57.

52. The ſeveral particulars reſpecting annuities at compound intereſt may more readily be found by the help of the three following tables, wherein are calculated the amount and preſent value of L. 1 annuity for years, at ſeveral rates of compound intereſt.

TABLE IV. Shewing the Amount of L. 1 Annuity for Years.

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.
1	1.	1.	1.	1.	1.	1.
2	2.03	2.035	2.04	2.045	2.05	2.06
3	3.0909	3.10623	3.1216	3.13703	3.1525	3.1836
4	4.18363	4.21494	4.24646	4.27819	4.31013	4.37460
5	5.30913	5.36246	5.41632	5.47071	5.52563	5.63709
6	6.46341	6.55015	6.63297	6.71689	6.80191	6.97532
7	7.66246	7.77941	7.89829	8.01915	8.14201	8.39384
8	8.89233	9.05168	9.21423	9.38001	9.54911	9.89747
9	10.15911	10.36849	10.58279	10.80211	11.02656	11.49131
10	11.46388	11.73139	12.00611	12.28821	12.57789	13.18079
11	12.80779	13.14199	13.48635	13.84118	14.20678	14.97164
12	14.19203	14.60196	15.02580	15.46403	15.91712	16.86994
13	15.61779	16.11303	16.62684	17.15991	17.71298	18.88214
14	17.08632	17.67698	18.29191	18.93211	19.59863	21.01506
15	18.59891	19.29568	20.02358	20.78405	21.57856	23.27597
16	20.15688	20.97103	21.82453	22.71933	23.65749	25.67253
17	21.76158	22.70501	23.69751	24.74171	25.84036	28.21288
18	23.41443	24.49969	25.64541	26.85508	28.13238	30.90565
19	25.11687	26.35718	27.67123	29.06356	30.53900	33.75999
20	26.87037	28.27968	29.77810	31.37142	33.06595	36.78559
21	28.67648	30.26947	31.96920	33.78313	35.71925	39.99273
22	30.53678	32.32890	34.24797	36.30338	38.50521	43.39229
23	32.45288	34.46041	36.61789	38.93703	41.43047	46.99583
24	34.42647	36.66653	39.08260	41.68919	44.50199	50.81557
25	36.45926	38.94985	41.64591	44.56521	47.72709	54.86451
26	38.55304	41.31310	44.31174	47.57064	51.11345	59.15638
27	40.70963	43.75906	47.08421	50.71132	54.66913	63.70576
28	42.93092	46.29063	49.96758	53.99333	58.40258	68.52811
29	45.21885	48.91080	52.96628	57.42303	62.32271	73.63979
30	47.57541	50.62267	56.08494	61.00707	66.43884	79.05818
31	50.00268	54.42947	59.32833	64.75238	70.76079	84.86167
32	52.50276	57.33450	62.70147	68.66624	75.29883	90.88977
33	55.07784	60.34121	66.20953	72.75623	80.06377	97.34316
34	57.73017	63.45315	69.85791	77.03025	85.06696	104.18375
35	60.46208	66.67401	73.65222	81.49662	90.32031	111.43478
36	63.27594	70.00760	77.59831	86.16396	95.83632	119.12086
37	66.17422	73.45787	81.70224	91.04134	101.62814	127.26812
38	69.15945	77.02889	85.97033	96.13820	107.70954	135.90420
39	72.23423	80.72490	90.40915	101.46442	114.09502	145.05845
40	75.40126	84.55028	95.02551	107.03032	120.79977	154.76196
41	78.66330	88.50954	99.82654	112.84668	127.83976	165.04767
42	82.02320	92.60737	104.81959	118.92478	135.23175	175.95053
43	85.48390	96.84863	110.01238	125.27640	142.99333	187.50756
44	89.04841	101.23816	115.41287	131.91384	151.14300	199.75801
45	92.71986	105.78167	121.02939	138.84996	159.70015	212.74349
46	96.50146	110.48403	126.87056	146.09821	168.68516	226.50810
47	100.39650	115.35097	132.94539	153.67263	178.11942	241.09858
48	104.40839	120.38825	139.26320	161.58790	188.02539	256.56449
49	108.54065	125.60184	145.83373	169.85935	198.42666	272.95836
50	112.79686	130.99791	152.66708	178.50303	209.34799	290.33586

Table IV. Shews the amount of L. 1 annuity; and is conſtructed by Equation 1ſt, 44; whereby $N=1\times\frac{R^t-1}{r}$. Or thus, ſeeing

1	$=a=$	amount for 1 year.
$a+R$	$=b=$	for 2 years.
$b+R^2$	$=c=$	for 3 years.
$c+R^3$	$=d=$	for 4 years, &c.

Hence, to L. 1, the firſt year of this table, add the firſt year of Tab. II. their ſum is the ſecond year of this table, to which add the ſecond year of Tab. II. their ſum will be the third year of this table, &c. From the nature of the table, $1 : N :: a : Na=s$, the amount of any annuity a, for the time and at the rate correſponding to N. Hence the following

TABLE.

Given.		
$a, t, r,$	$s=Na.$	r being given, t is found oppoſite to N, and t being given, r is found above N in the Table.
$s, t, r,$	$a=\frac{s}{N}.$	
$s\ t, a,$ / $s, r, a,$	$N=\frac{s}{a}.$	

Ex. 1*ſt.* Given, $a=$L. 80, $t=20$ years, and the rate 4 per cent.; ſought the amount. Here,

$Na=29.7781\times 80=$L. 2382.248, the amount.

Ex

Ex. 2d. Given, s=L. 1000, t=15 years, and the rate of intereſt 5 per cent.; ſought, the annuity.

$$\frac{s}{N} = \frac{1000}{21.57856} = \text{L. } 46.3423, \text{ the annuity.}$$

Ex. 3d. Given, a=L. 48.114, s=L. 1000, and t=15 years; ſought, the rate of intereſt.

$\frac{s}{a} = \frac{1000}{48.114} = 20.7840 = N$ for 15 years, correſponding to $4\frac{1}{2}$ per cent.

Ex. 4th, Given a = L. 5.25, s = L. 419.297, and the rate $4\frac{1}{2}$ per cent.; ſought, the time of continuance.

$\frac{s}{a} = \frac{419.297}{5.25} = 79.8661 = N$ for $4\frac{1}{2}$ per cent. correſponding to 34 years; and for the odd days ſay,

81.49662	79.86610
77.03025	77.03025

as 4.46637 : 1 :: 2.83585 : .635 = 232 days.

TABLE

TABLE V. Shewing the present Worth of L. 1 Annuity for Years.

Yrs.	3 per cent.	3½ pr cent.	4 per cent.	4½ pr cent.	5 per cent.	6 per cent.
1	0.97087	0.96618	0.96154	0.95694	0.95238	0.94339
2	1.91347	1.89969	1.88609	1.87267	1.85941	1.83339
3	2.82861	2.80164	2.77509	2.74896	2.72325	2.67301
4	3.71710	3.67308	3.62989	3.58752	3.54595	3.46510
5	4.57971	4.51505	4.45182	4.38997	4.32947	4.21236
6	5.41719	5.32855	5.24214	5.15787	5.07569	4.91732
7	6.23028	6.11454	6.00205	5.89270	5.78637	5.58238
8	7.01969	6.87395	6.73274	6.59588	6.46321	6.20979
9	7.78611	7.60768	7.43533	7.26879	7.10782	6.80169
10	8.53020	8.31660	8.11089	7.91272	7.72173	7.36008
11	9.25262	9.00155	8.76047	8.52892	8.30641	7.88687
12	9.95400	9.66333	9.38507	9.11858	8.86325	8.38384
13	10.63495	10.30274	9.98565	9.68285	9.39357	8.85268
14	11.29607	10.92052	10.56312	10.22282	9.89864	9.29498
15	11.93793	11.51741	11.11838	10.73954	10.37966	9.71225
16	12.56110	12.09412	11.65229	11.23401	10.83776	10.10589
17	13.16612	12.65132	12.16567	11.70719	11.27406	10.47726
18	13.75351	13.18968	12.65929	12.15999	11.68958	10.82760
19	14.32380	13.70983	13.13394	12.59329	12.08532	11.15812
20	14.87747	14.21240	13.59032	13.00793	12.46221	11.46992
21	15.41502	14.69797	14.02916	13.40472	12.82115	11.76407
22	15.93691	15.16712	14.45111	13.78442	13.16300	12.04158
23	16.44360	15.62041	14.85684	14.14777	13.48857	12.30338
24	16.93554	16.05836	15.24696	14.49548	13.79864	12.55036
25	17.41315	16.48151	15.62208	14.82821	14.09394	12.78335
26	17.87684	16.89035	15.98276	15.14661	14.37518	13.00316
27	18.32703	17.28536	16.32958	15.45130	14.64303	13.21053
28	18.76411	17.66702	16.66306	15.74287	14.89813	13.40616
29	19.18845	18.03576	16.98371	16.02189	15.14107	13.59072
30	19.60044	18.39204	17.29203	16.28888	15.37245	13.76483
31	20.00043	18.73627	17.58849	16.54439	15.59281	13.92910
32	20.38876	19.06886	17.87355	16.78889	15.80267	14.08404
33	20.76579	19.39020	18.14764	17.02286	16.00255	14.23023
34	21.13183	19.70068	18.41119	17.24676	16.19290	14.36814
35	21.48722	20.00066	18.66461	17.46102	16.37419	14.49824
36	21.83225	20.29049	18.90828	17.66604	16.54685	14.62098
37	22.16723	20.57052	19.14258	17.86224	16.71128	14.73678
38	22.49246	20.84108	19.36786	18.04999	16.86789	14.84602
39	22.80821	21.10250	19.58448	18.22965	17.01704	14.94907
40	23.11477	21.35507	19.79277	18.40158	17.15908	15.04629
41	23.41240	21.59910	19.99305	18.56611	17.29436	15.13802
42	23.70136	21.83488	20.18562	18.72355	17.42320	15.22454
43	23.98190	22.06269	20.37079	18.87421	17.54591	15.30617
44	24.25427	22.28279	20.54884	19.01838	17.66277	15.38318
45	24.51871	22.49545	20.72004	19.15634	17.77406	15.45583
46	24.77545	22.70092	20.88465	19.28837	17.88006	15.52437
47	25.02471	22.89943	21.04293	19.41471	17.98101	15.58902
48	25.26671	23.09124	21.19513	19.53560	18.07715	15.65002
49	25.50166	23.27656	21.34147	19.65130	18.16872	15.70757
50	25.72976	23.45562	21.48218	19.76201	18.25592	15.76186

Table V. Shews the preſent value of L. 1 annuity, and is conſtructed by Equation 11ſt, 47. whereby $N=1\times\frac{1-\frac{1}{R^t}}{r}$=the number of years purchaſe an annuity is worth: Or thus, Seeing

$\frac{1}{R}=a$=Preſent worth for 1 year.

$a+\frac{1}{R^2}=b=$ for 2 years.

$b+\frac{1}{R^3}=c=$ for 3 years.

$c+\frac{1}{R^4}=d=$ for 4 years, &c.

Hence the firſt year in this table and in table III. are the ſame, the ſum of the firſt year in this and of the ſecond in Table III. makes the ſecond in this, and the ſum of the ſecond in this, and of the third in Table III. makes the third year in this table, &c. From the nature of the table, $1:N::a:Na=p$, the preſent worth of any annuity a, for the time, and at the rate correſponding to N. Hence the following

TABLE.

Given.	
$a, t, r,$	$p=Na$
$p, t, r,$	$a=\frac{p}{N}$
$p, t, a,$ / $p, r, a,$	$N=\frac{p}{a}$

Ex. 1ſt, Given a=L. 50, t=15 years, and the rate 5 per cent.; ſought the preſent worth. Here,

$$Na=10.37966\times 50=\text{L. } 518.983=p.$$

Ex.

Ex. 2d, Given, $p=270$, $t=59$, and the rate 3 per cent.; ſought, the annuity. Here,

$$\frac{p}{N}=\frac{270}{25.729764}=\text{L. } 10.493685, \text{ the annuity.}$$

Ex. 3d, Given, $a=$L. 100, $p=$L. 1037.966, and $t=15$ years; ſought, the rate of intereſt.

$\frac{p}{a}=\frac{1037.966}{100}=10.37966=N$ for 15 years, correſponding to 5 per cent.

Thus, as far as the tables extend, may theſe difficult equations be reſolved, where R is ſought in Equation 3d, 44. and 47. and where it is involved to a high power.

TABLE VI. Shewing the Annuity which L. 1 will purchaſe.

Yrs.	3 per cent	3½ per cent.	4 per cent.	4½ per cent.	5 per cent	6 per cent.
1	1.03	1.035	1.04	1.045	1.05	1.06
2	.522610	.526400	.530196	.533997	.537805	.545437
3	.353530	.356934	.360348	.363773	.367208	.374109
4	.269027	.272251	.275490	.278743	.282011	.288591
5	.218354	.221481	.224627	.227791	.230974	.237396
6	.184597	.187668	.190762	.193878	.197016	.203362
7	.160506	.163544	.166609	.169701	.172820	.179135
8	.142456	.145476	.148527	.151609	.154722	.161036
9	.128434	.131446	.134493	.137574	.140690	.147022
10	.117230	.120241	.123291	.126378	.129504	.135868
11	.108077	.111092	.114149	.117248	.120389	.126793
12	.100462	.103484	.106552	.109666	.112825	.119277
13	.094029	.097061	.100143	.103275	.106455	.112960
14	.088526	.091570	.094669	.097820	.101024	.107584
15	.083766	.086825	.089941	.093114	.096342	.102962
16	.079611	.082684	.085820	.089015	.092269	.098952
17	.075932	.079043	.082198	.085417	.088699	.095444
18	.072708	.075816	.078993	.082237	.085546	.092356
19	.069813	.072940	.076138	.079407	.082745	.089620
20	.067215	.070361	.073582	.076876	.080242	.087184
21	.064871	.068036	.071280	.074600	.077996	.085004
22	.062747	.065932	.069198	.072545	.075970	.083045
23	.060813	.064018	.067309	.070682	.074136	.081278
24	.059047	.062273	.065586	.068987	.072471	.079679
25	.057427	.060674	.064012	.067439	.070952	.078226
26	.055938	.059205	.062567	.066021	.069564	.076904
27	.054564	.057852	.061238	.064719	.068292	.075697
28	.053293	.056602	.060013	.063521	.067122	.074592
29	.052114	.055445	.058879	.062414	.066045	.073579
30	.051019	.054371	.057830	.061391	.065051	.072649
31	.049999	.053372	.056855	.060443	.064132	.071752
32	.049046	.052441	.055948	.059563	.063280	.071002
33	.048156	.051572	.055104	.058744	.062490	.070273
34	.047322	.050759	.054315	.057982	.061755	.069598
35	.046539	.049998	.053577	.057270	.061072	.068974
36	.045804	.049284	.052887	.056606	.060434	.068395
37	.045111	.048613	.052239	.055984	.059840	.067857
38	.044459	.047982	.051632	.055402	.059284	.067358
39	.043844	.047387	.051061	.054857	.058764	.066894
40	.043262	.046827	.050523	.054343	.058278	.066462
41	.042712	.046298	.050017	.053861	.057822	.066059
42	.042191	.045798	.049540	.053408	.057395	.065683
43	.041698	.045325	.049090	.052982	.056993	.065333
44	.041230	.044877	.048664	.052581	.056616	.065006
45	.040785	.044453	.048262	.052202	.056262	.064701
46	.040362	.044051	.047882	.051845	.055928	.064415
47	.039960	.043669	.047522	.051507	.055614	.064148
48	.039577	.043306	.047181	.051188	.055318	.063927
49	.039213	.042962	.046857	.050887	.055039	.063664
50	.038865	.042634	.046550	.050602	.054777	.063444

Table VI. ſhews the annuity which L. 1 will purchaſe, and is conſtructed by Equation 2d, 47. whereby $N=\frac{r}{1-\frac{1}{R^t}}$. Or thus, it being the reciprocal of Table V. N of Table VI. $=\frac{1}{N}$ of Table V. From the nature of the table, $1 : N :: p : Np=a$, the annuity which p will purchaſe, for the time and at the rate correſponding to N. Hence the following

TABLE.

$p, t, R,$	$a = Np.$
$a, t, R,$	$p = \frac{a}{N}.$
$p, t, a,$	$N = \frac{a}{p}.$
$p, R, a,$	

Ex. 1ſt, Given, $p=$L. 1000, $t=$ 20 years, and the rate 5 per cent.; ſought, the annuity. Here,

$Np=.080242\times 1000=$L. 80.242, annuity.

Ex. 2d, Given, $a=$L. 30, $t=19$ years, and the rate 4 per cent.; ſought, the preſent worth. Here,

$\frac{a}{N}=\frac{30}{.076138}=$L. 394.021, the preſent worth.

Ex. 3d, Given, $a=$L. 80.242, $p=$L. 1000, and $t=20$ years; ſought, the rate of intereſt. Here,

$\frac{a}{p}=\frac{80.242}{1000}=.080242=N$ for 20 years, correſponding to 5 per cent. the rate ſought.

More Examples.

1ſt, A gentleman purchaſes an eſtate for L. 1000, and is allowed to retain the purchaſe-money in his own hands, upon condition of paying to the former proprietor 4 per cent. per annum: He ſets the eſtate at ſuch a yearly rent as to have 6 per cent for his money; ſought, the time in which the eſtate will clear itſelf.

Here the purchaſer's yearly profit-rent is 2 per cent. upon the purchaſe-money, and the time is ſought in which this intereſt will equal the principal.——By Remark 3d, 43,

$$s = 2p \text{ when } t = \frac{L.2}{L.1.02} = \frac{.30103}{.0086} = 34.886 \text{ years.}$$

2d, A puts L. 1500 out at intereſt, B hath an annuity of L. 75 to continue 50 years; which of them will amount to the greateſt ſum at the end of the 50 years; and what is the preſent worth of the difference, at the rate of $4\frac{1}{2}$ per cent. per annum? Here, by Table II. and IV.

The amount of L. 1500 for 50 years at $4\frac{1}{2}$ per cent.	= L. 13548.945
of L. 75 annuity for 50 years	= 13387.727
A's amount exceeds that of B by	161.218

The preſent worth of which by Table III. = L. 17.848.

3d, A hath a term of 7 years in an eſtate of L. 50 per annum; B hath a reverſion of the ſame eſtate of 14 years after A; and C hath a further reverſion of 21 years after A and B; ſought, the preſent worths of the ſeveral terms, reckoning intereſt at $4\frac{1}{2}$ per cent. Here $a \times \frac{1 - \frac{1}{R^t}}{rR^n}$ = their preſent worths reſpectively. Or thus,

By

By Table V. 42 years = 18.72355 × 50 = 936.1775
21 years = 13.40472 × 50 = 670.236
7 years = 5.89270 × 50 = 294.635

ſubtracting theſe from each preceding term, we have

A's term = L. 294.635, B's = L. 375.601, C's = L. 265.9415.

4th, A perſon having 7 years to run in a leaſe of an eſtate of L. 80 per annum, it is required what ſum he ought to pay at preſent to have the leaſe renewed by having 20 years added thereto, reckoning intereſt at 6 per cent.? Here,

$a \times \frac{1 - \frac{1}{R^{20}}}{rR^{7}} = 80 \times 7.62815 = \text{L. } 610.25 = p$. Or thus,

p, of L. 1 annuity for 27 years = 13.21053
for 7 years = 5.58238

7.62815 × 80 =
L. 610.25, = the ſum he ought to pay.

5th, A leaſe of an eſtate, to continue 14 years, is offered for L. 250 fine, and L. 44 yearly rent; but the tenant wants to reduce the rent to L. 20 per annum; what ought the fine to be at 6 per cent? Here,

44—20 = 24, and Tab.V. 9.29498 × 24 = 223.0795
+ 250.

The whole fine = L. 473.0795

6th, The proprietor of an eſtate, having, 20 years ago, granted the leaſe of a farm for 38 years, is deſirous of recovering the ſame: It is required to find what he ought to pay for the recovery of

ſaid

ſaid leaſe, upon the ſuppoſition that the farm is worth L. 40 of yearly additional rent, and reckoning intereſt at 5 per cent. Here are given, a=L. 40, t=18, and the rate 5 per cent. ſought p.

By Tab. V. 11.6896×40=L. 467.584=p.

53. *Of Annuities computed at Compound Intereſt, when the firſt Payment is either due, or paid per Advance.*

1ſt, To find the amount of an annuity in arrears, the firſt payment being due per advance, at a given rate of compound intereſt. Here, as the firſt year's annuity (a) bears intereſt, the laſt year's annuity will be repreſented by aR^t; and deriving a ſeries from the quantity aR, in the *ratio* of 1 : R, we ſhall have $aR+aR^2+aR^3\ldots+aR^t=s$; hence $s=a\times\frac{R^{t+1}-R}{r}=a\times\frac{R^t-1}{r}\times R=aN\times R$ of Table IV.

Ex. It is required, to find the amount of an annuity of L. 5, at the end of 30 years, the firſt payment being due per advance, and reckoning intereſt at 4 per cent. Here,

$a\times\frac{R^{t+1}-R}{r}=5\times\frac{2.333713}{.04}$=L. 291.64, the amount.

If the ſeries expreſſing the amount of an annuity, when the firſt payment is due at the end of the year, (44.) be ſubtracted from the above ſeries, their difference will be $a\,R^t-a$, in favour of the latter, which, of conſequence, will exceed the former, by the accumulated intereſt of one year's annuity for the time of its continuance. Or thus: The amount of an annuity for t years, when the firſt payment is due from the beginning of the year,

year, will be equal to its amount for $t+1$ years—a, when the firſt payment is due at the end of the year.

2d, To find the preſent value of an annuity, the firſt payment being made *per* advance, at a given rate of compound intereſt. Here we ſhall have,

$a+\frac{a}{R}+\frac{a}{R^2}\ldots+\frac{a}{R^{t-1}}=p$, the preſent value of the annuity, which gives $p=a\times\frac{R-\frac{1}{R^{t-1}}}{r}=a\times\frac{1-\frac{1}{R^t}}{r}\times R=aN\times R$ of Table V.

Ex. It is required, to find the preſent value of an annuity of L. 20, to continue 19 years, the firſt payment being made *per* advance, and reckoning intereſt at 4 per cent. Here,

$$a\times\frac{R-\frac{1}{R^{t-1}}}{r}=20\times\frac{.5464}{.04}=\text{L. } 273.4, \text{ the value.}$$

If the ſeries expreſſing the preſent value of an annuity, when the firſt payment is made at the end of the year, (47.) be ſubtracted from the above ſeries, their difference will be $a-\frac{a}{R^t}$, in favour of the latter, which, of conſequence, will exceed the former by the difference betwixt the annuity and the preſent value of one payment thereof due at the end of t years. Or thus: The preſent value of an annuity for t years, when the firſt payment is made per advance, will be equal to its preſent value for $t-1$ years$+a$, when the firſt payment is due at the end of the year.

Remarks

Remarks upon Annuities at Compound Intereſt.

54. 1ſt, The expreſſion $\frac{1-\frac{1}{R^t}}{r}$, in Equation 1ſt, 47. is always leſs than $\frac{1}{r}$, except when t, the time, is indefinite; of conſequence, the years purchaſe of an annuity for a certain number of years, at compound intereſt, are always leſs than the years purchaſe of a perpetuity: Hence compound intereſt, by which the value of a perpetuity is determined, is the proper ſtandard for aſcertaining the value of an annuity.

2d, The time in which the preſent value of an annuity is equal to its reverſion may be found by the equation $\frac{a}{r}-\frac{a}{rR^t}=\frac{a}{rR^t}$, which gives $\frac{a}{r}=\frac{2a}{rR^t}$.

$1=\frac{2}{R^t}$.

$R^t=2$, and

$t=\frac{L.\,2}{L.\,R}=$ the time in which a ſum of money doubles itſelf at a given rate of compound intereſt.

3d, Beſides the two methods mentioned in the conſtruction of Tables IV. and V. the amount and preſent value of an annuity may alſo be calculated thus:

1ſt, To find the amount of L. 1 annuity for years.

Let $1=a=$ the amount for 1 year.

Then $aR+1=b=$ 2.

$bR+1=c=$ 3.

$cR+1=d=$ 4 years, &c.

2d,

2d, To find the preſent value of L. 1 annuity for years.

Let $\frac{1}{R}=a=$ the preſent value for 1 year.

Then $\frac{a+1}{R}=b=$ 2.

$\frac{b+1}{R}=c=$ 3.

$\frac{c+1}{R}=d=$ 4 years, &c.

SUPPLEMENT *to* CHAPTER VI.

Of a Sinking Fund to extinguiſh the National Debt.

55. WHEN a debtor's funds are inſufficient to pay the intereſt of his debt, this muſt increaſe yearly by the accumulation of the deficient intereſt; but if he can ſpare a little, after paying this intereſt, that little may, with proper œconomy, become a ſinking fund to extinguiſh his debt in time.

Any part of the national debt is ſaid to be bought at par, when the legal intereſt of the purchaſe money, at 5 per cent. is equal to the intereſt of the ſtock purchaſed, or when the purchaſer hath 5 per cent for his money.

Thus, pr cents. L. 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6,
Pars, L. 60, 70, 80, 90, 100, 110, 120.

As a debtor hath it always in his power to diſcharge his debt, by paying the ſum borrowed, when he is able, and is ſo diſpoſed, Government, it is preſumed, is not obliged to pay above the par of 5 per cent for each L. 100 of national debt.——

In the following particulars, units are aſſumed in place of millions, when the national debt is concerned; which debt may conſiſt either of money borrowed at 3, 4, or 5 per cent. or in the value of annuities paid by Government, for a certain number of years, at 5 or 6 per cent.; and may be in all about L. 262, at $3\frac{1}{2}$ per cent. *anno* 1792; at which period there was, in the ſpace of ſix years, the ſum of L. 8 recovered, and the national ſinking fund was raiſed to L. 1.4, which is ſuppoſed to accumulate till it riſes to L. 4.

1ſt, It is required, to find the time in which a ſinking fund of L. 1.4 will extinguiſh L. 25—8 = L. 17 of 3 per cents, the purchaſes being made at the market-price of 90.

Here, 100 : 90 : : 17 : 15.3; and, by Table IV. $\frac{s}{a} = \frac{15.3}{1.4} = 10.928 = N$, at 3 per cent. correſponding to 9.59 years, the time required; being that period at which Government can, if it ſhall be thought proper, point all the artillery of the ſinking fund againſt the 4 and 5 per cents, for the reduction of the ſame.

2d, It is required, to find the time in which the national ſinking fund will riſe to L. 4; beyond which it is ſuppoſed it needs not to accumulate, being then ſufficient to anſwer all the purpoſes of its deſtination.

On or before the year 1808, to which period the long annuities of 1778 extend, it is preſumed that L. 12 of annuities, at 5 per cent. will ceaſe; whoſe intereſt = L. .60.

Therefore, to the preſent ſinking fund = 1.40
Add the intereſt of L. 17 recovered in 9.59 years = 0.51

Their ſum = L. 1.91

And

And find the capital of 4 per cents, which a ſinking fund of L. 1.91 will recover in 16 — 9.59 = 6.41 years. $Na = 7.152 \times 1.91 =$ L. 13.66, whoſe intereſt, at 4 per cent. = .546
to which add 1.91 + .60 = 2.510

Their ſum is the ſinking fund, an. 1808 = 3.056

Again, 4 — 3.056 = .944; and $\frac{944}{04}$ = L. 23.6, a capital of 4 per cents, which L. 3.056 will recover in the ſpace of 7 years. Hence the national ſinking fund will riſe to L. 4 anno 1815.

3d, It is required, to find the amount of the national debt diſcharged when the ſinking fund riſes to L. 4.

Prior to an. 1792, there was redeemed, in 6 yrs, a ſum	= L.	8.00
From hence, in the ſpace of 9.59 years, a ſum	=	17.00
During 6 41 years, prior to anno 1808,	=	13.66
In anno 1808, of annuities,	=	12.00
During ſeven years, prior to anno 1815,	=	23.60
In all,	L.	74.26

That a ſinking fund may have its proper effect, it ought to accumulate yearly, with the intereſt of the capital which it hath recovered. But when it is limited, for inſtance, to L. 4, the nation may be relieved from taxes to the amount of ſaid intereſt.

4th, It is required, to find the time in which a ſinking fund of L. 4, without accumulation, will reduce the national debt to the ſame ſtate in which it was anno 1755, the medium rate of intereſt being $3\frac{1}{2}$ per cent. and the purchaſes made at 100.

Let L. 9.45 be the intereſt of the national debt, anno 1786, correſponding to a capital of L. 270,

at $3\frac{1}{2}$ per cent. Upon this ſuppoſition:

To the amount of debt anno 1755,	= L.	72.00
Add of capital recovered in 29 years prior to anno 1815, as above, a ſum =		74.26
And of annuities expiring,	=	10.00
Their ſum	= L.	156.26

Hence $270 - 156.26 = L.\ 113.74$, the debt to be extinguiſhed; ſought, the time. Here, $\frac{113.74}{4} = 28.43$ years, the time required, extending to the year 1844: at which period the national taxes, including one million of a ſinking fund, will be $10.45 - 2.52 = 7.93$ millions leſs, yearly, than they were anno 1786.

5th, It is required, to find the effect which a ſinking fund of L. 1.4 will have upon a debt of L. 270, in the ſpace 25 years, reckoning intereſt at $3\frac{1}{2}$ per cent.; and what will be the amount of ſaid fund at the expiration of that period.

By Table IV. $Na = 38.95 \times 1.4 = L.\ 54.53$, the debt diſcharged in 25 years; and $1.4 + 1.9 = L.\ 3.3$, the ſinking fund.

6th, It is required, to find the time in which a ſinking fund of L. 1.4 will extinguiſh a debt of L. 270, reckoning intereſt at $3\frac{1}{2}$ per cent. Here,

By Equation 4. $44.\frac{L.\frac{n}{a}+1}{L.R} = \frac{.8893017}{.0149403} = 59.52$ years, the time required.

CHAP.

CHAP. VII.

Of the Value of a Perpetuity, or Freehold Eſtate.

56. IN treating of a perpetuity or freehold-eſtate; let s repreſent the value, and a the yearly rent thereof; then the progreſſion indefinitely continued, viz. $\frac{a}{R}+\frac{a}{R^2}+\frac{a}{R^3}+\frac{a}{R^4}$, &c. $= s$. (47.) which progreſſion, having no laſt term, gives $s=\frac{a}{R-1}=\frac{a}{r}$. (40.) from whence all the varieties in the buying and ſelling of eſtates in fee-ſimple may be reſolved, as in the following

TABLE.

Given.		Sought s.	
a, R,	$s=\frac{a}{R-1}=\frac{a}{r}$.	38.	$s=\overline{s-\frac{a}{R}}\times R$.
s, r,	$a=rs$.		$s=Rs-a$.
s r.	$r=\frac{a}{s}$ & $R=1+\frac{a}{s}$		$\overline{R-1}\times s=a$.
			Or thus:
		40.	$s=\frac{R\times\frac{a}{R}}{R-1}=\frac{a}{R-1}$.

Let

Let N repreſent the number of years purchaſe a is worth, then $Na=s$; and for a ſubſtituting its value, rs, from the fee-ſimple equation, we ſhall have $Nrs=s$; which gives $Nr=1$, $N=\frac{1}{r}$ & $r=\frac{1}{N}$. Or thus: Seeing $s=Na$, and a, the rent, may be conſidered as the intereſt drawn yearly for the purchaſe-money, hence $s:a=Na:a=N:1=1:r$, which give $s=Na$, $rs=a$, and $Nr=1$. from whence the following

EQUATIONS.

$$s=Na=\frac{a}{r}.$$

$$a=\frac{s}{N}=rs.$$

$$N=\frac{s}{a}=\frac{1}{r}.$$

$$r=\frac{a}{s}=\frac{1}{N}.$$

Ex. 1ſt, Sought, the value of a freehold eſtate, whoſe yearly rent is L. 150, diſcounting intereſt at $3\frac{1}{2}$ per cent. Here $\frac{a}{r}=\frac{150}{.035}=$ L. $4285.7=s$, the value. Had the purchaſe been made at 28.57 years purchaſe, correſponding to $3\frac{1}{2}$ per cent. then $150\times 28.57=$L. $4285.5=s$, the value.

Ex. 2d, A gentleman purchaſes an eſtate for L. 14,000; at what yearly rent muſt he let it, to have 4 per cent. upon his purchaſe-money? Here, $rs=.04\times 14000=$L. $560=a$.

Ex. 3d, An eſtate, which coſt L. 8000, is let in tack

tack at the rent of L. 360 a-year; ſought, at what number of years purchaſe it is bought, and what rate of intereſt the purchaſer hath for his money? Here $\frac{1}{a}=\frac{8000}{360}=22.2\dot{2}=N$; and $\frac{a}{1}=\frac{360}{8000}=.045=r=4\frac{1}{2}$ per cent.

Of a Reverſion in Perpetuity.

57. A reverſion in perpetuity, or in fee-ſimple, being the ſame with the reverſion of an annuity when the time prior to its commencement is equal to the duration of the annuity, its value may be found thus: Divide the value of the perpetuity, viz. $\frac{a}{r}$ by R^t, t being the time prior to its commencement, the quotient gives the reverſion: Thus $\frac{a}{r}R^t=s$, the reverſion in perpetuity. Hence the following

TABLE.

Given,		
$a, t, R,$	$s=\frac{a}{rR^t}$.	Equ. 1.
$s, t, R,$	$a=rs\times R^t$.	2.
$s, t, a,$	$R^{t+1}-R^t=\frac{a}{s}$.	3.
$s, R, a,$	$t=\frac{l.a-L.rs}{L.R}$.	4.

Ex. 1ſt, Sought, the preſent value of a reverſion in perpetuity, which is to commence 40 years hence, and whoſe yearly rent is L. 70, diſcounting intereſt at 4 per cent. Here,

$\frac{a}{rR^t}=\frac{70}{.19204}=$ L. 364.5, the value ſought.

Ex.

Ex. 2d, There is a reverſion in fee-ſimple ſold for L. 500, which is to commence 25 years hence; ſought, the yearly rent thereof, diſcounting intereſt at 5 per cent.

$rs \times R^t = 25 \times 3.3863 = 84.6575$, the rent ſought.

Ex. 3d, The reverſion of an annuity of L. 20 a-year is ſold for L. 222.735; ſought, the duration of the annuity, reckoning intereſt at 5 per cent. Here,

$$\frac{L.a - L.rs}{L.R} = \frac{.2542716}{.0211893} = 12 \text{ years.}$$

Ex. 4th, There is an annuity of L. 35, to continue 19 years, whoſe reverſion is ſold for L. 256.9 ready money; ſought, the rate of intereſt allowed the purchaſer. Here,

$\frac{a}{r} = \frac{35}{256.9} = .13624 = R^{20} - R^{19}$, found by Table II. betwixt 5 and 6 per cent.; therefore take two poſitions, &c.

1ſt Poſit. $R = 1.05$ Reſult 277.652 Error + 20.744
2d Poſit. $R = 1.055$ Reſult 230.095 Error — 26.813
By the queſtion, Reſult 256.9 Diff. of Reſult of 1ſt & 2d = 47.557

Then 47.557 : .005 : : 20.744 : .00218 to be added to the 1ſt poſition, making the rate 5.218 per cent. nearly.

Ex. 5th, Which is moſt advantageous, a term of 16 years in an eſtate of L. 50 per annum; or the reverſion of the ſame eſtate for ever, after the expiration of the ſaid term, reckoning intereſt at 4 per cent.? Here,

By Equ. 1ſt, the Reverſion = 13.3477 × 50 = 667.385
By Table V. the Annuity = 11.11838 × 50 = 555.919

The Reverſion is better than the Term of 16 years by L. 111.466

Ex.

Ex. 6th, A and B purchaſe an eſtate for L. 6700, which, at the term of Martinmas 1790, they agree to divide into two equal parts.

A ſets his part for the yearly rent of L. 180, and has, beſides, timber, and an old manſion-houſe, valued at L. 260.

B's part is under tack for 11 years to run, at the yearly rent of L. 143, and the timber upon it is valued at L. 22.

Sought, firſt, At what rate of intereſt the purchaſe was made; ſecondly, How much A's part is better than B's; thirdly, At what yearly rent muſt B ſet his part, 11 years hence, to make it equal to A's in its preſent value, reckoning intereſt at 5 per cent.

$$\frac{a}{s} = \frac{180}{3090} = .05825 = \text{L. } 5 : 16 : 6 \text{ per cent.}$$

A's Part $= \frac{a}{r} = \frac{180}{.05} = 3600$	B's Part $= \frac{a}{r} = \frac{143}{.05} = 2860$
His timber and houſe $=$ 260	His timber $=$ 22
A's value $=$ L. 3860	B's value $=$ L. 2882

So that A exceeds B in value L. 978. By 57. Equ. 2d, $\frac{978.00}{11.6935} = \text{L. } 83.636 = $ B's riſe of rent at the end of 11 years, to make his part equal in preſent value to that of A.

Remarks upon the Purchaſe of a Perpetuity.

58. 1ſt, A freehold-eſtate or perpetuity, purchaſed at the rate of intereſt r, is bought at a number of years purchaſe $= \frac{1}{r}$, and the rate of intereſt r, at which the purchaſe-money of a freehold eſtate is valued for N years purchaſe $= \frac{1}{N}$: So that N and r are reciprocal of, and diſcover one another. Thus:

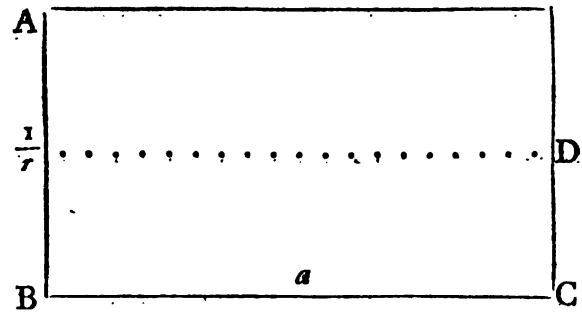
If $N=35$, then $r=.02857=2.857$ per cent.

Or, If $r=.04$, then $N=25$ years purchaſe.

2d, N, the number of years purchaſe at which a perpetuity is bought or ſold at a given rate of compound intereſt, is always equal to the number of years in which money doubles itſelf at the ſame rate of ſimple intereſt, ſeeing either of them $=\frac{1}{r}$, (23. & 56.) Thus:

When a purchaſe is made at	per cent.	then $N=$	years purchaſe.
$2\frac{1}{2}$		40	
3		33.3	
$3\frac{1}{2}$		28.57	
4		25	
$4\frac{1}{2}$		22.2	
5		20	
6		16.6	
8		12.5,	and *vice verſa*.

3d, The value of a perpetuity may be expreſſed by the area of a right-angled parallelogram AC,



whoſe breadth $AB=\frac{1}{r}$, and its length $BC=a$ the rent of the eſtate; for $AB\times BC=\frac{a}{r}=s$, the value of the perpetuity; and half of the parallelogram, viz,

viz. $BD=\frac{a}{2r}$=preſent value, or the reverſion of an annuity a, whoſe duration is equal to the time in which a ſum of money will double itſelf at the rate r of compound intereſt: For in Equation 4th, 47. in place of p, put its value in this caſe, viz. $\frac{a}{2r}$, then $t=\frac{L.\,a-L.\,\frac{a}{2}}{L.\,R}=\frac{L.\,2}{L.\,R}$=the time in which a ſum of money will double itſelf at the rate r of compound intereſt.

4th, The method of buying and ſelling eſtates, at a certain number of years purchaſe, is very proper, being both ſimple, and well adapted to calculations upon the principles of compound intereſt, by which alone the value of a perpetuity can be aſcertained.

5th, The purchaſer of a perpetuity, if he can afford it, may venture to give, at leaſt, 10 years purchaſe more for an eſtate than what would ariſe from the legal intereſt of 5 per cent. on account of the ſuperior ſecurity, the chance of the riſe of ground in its value; and becauſe he is allowed compound intereſt upon the purchaſe-money, and other advantages.

A TABLE, Shewing the present Worth of L. 1 for Years.

Yrs.	3 per cent.	4 per cent.	5 per cent.	Yrs.	3 per cent.	4 per cent.	5 per cent.
51	.221463	.135301	.083051	71	.122619	.061749	.031301
52	.215013	.130097	.079096	72	.119047	.059374	.029811
53	.208750	.125093	.075330	73	.115580	.057091	.028391
54	.202670	.120282	.071743	74	.112214	.054895	.027039
55	.196767	.115656	.068326	75	.108945	.052784	.025752
56	.191036	.111207	.065073	76	.105772	.050754	.024525
57	.185472	.106930	.061974	77	.102691	.048801	.023357
58	.180070	.102817	.059023	78	.099700	.046924	.022245
59	.174825	.098963	.056212	79	.096796	.045120	.021186
60	.169733	.095060	.053536	80	.093977	.043384	.020177
61	.164789	.091404	.050986	81	.091240	.041716	.019216
62	.159990	.087889	.048858	82	.088582	.040111	.018301
63	.155330	.084508	.046246	83	.086002	.038569	.017430
64	.150806	.081258	.044044	84	.083497	.037085	.016600
65	.146413	.078133	.041946	85	.081065	.035659	.015809
66	.142149	.075128	.039949	86	.078704	.034287	.015056
67	.138009	.072238	.038047	87	.076412	.032968	.014339
68	.133989	.069460	.036235	88	.074186	.031700	.013657
69	.130086	.066788	.034509	89	.072027	.030481	.013006
70	.126297	.064219	.032866	90	.069928	.029309	.012387
				91	.067891	.028182	.011797

A TABLE Shewing the present Worth of L. 1 Annuity for Years.

Yrs.	3 per cent.	4 per cent.	5 per cent.	Yrs.	3 per cent.	4 per cent.	5 per cent.
51	25.9512	21.6174	18.3389	71	29.2460	23.4562	19.3739
52	26.1662	21.7475	18.4180	72	29.3650	23.5156	19.4037
53	26.3749	21.8726	18.4934	73	29.4806	23.5727	19.4321
54	26.5776	21.9929	18.5651	74	29.5928	23.6276	19.4592
55	26.7744	22.1085	18.6334	75	29.7018	23.6804	19.4849
56	26.9654	22.2198	18.6985	76	29.8076	23.7311	19.5094
57	27.1509	22.3267	18.7605	77	29.9102	23.7799	19.5328
58	27.3310	22.4295	18.8195	78	30.0099	23.8268	19.5550
59	27.5058	22.5284	18.8757	79	30.1067	23.8720	19.5762
60	27.6755	22.6234	18.9292	80	30.2007	23.9153	19.5964
61	27.8403	22.7148	18.9802	81	30.2920	23.9571	19.6156
62	28.0003	22.8027	19.0288	82	30.3805	23.9972	19.6339
63	28.1556	22.8872	19.0750	83	30.4665	24.0357	19.6514
64	28.3064	22.9685	19.1191	84	30.5500	24.0728	19.6680
65	28.4528	23.0466	19.1610	85	30.6311	24.1085	19.6838
66	28.5950	23.1218	19.2010	86	30.7098	24.1428	19.6988
67	28.7330	23.1940	19.2390	87	30.7862	24.1757	19.7132
68	28.8670	23.2635	19.2753	88	30.8604	24.2074	19.7268
69	28.9971	23.3302	19.3098	89	30.9324	24.2379	19.7398
70	29.1234	23.3945	19.3426	90	31.0024	24.2672	19.7522
				perpetuity		33.3333	25.0000	20.0000

A TABLE Shewing the Amount of L. 1 Annuity for Years.

Yrs.	3 per cent.	4 per cent.	5 per cent.	Yrs.	3 per cent.	4 per cent.	5 per cent.
51	117.181	159.774	220.815	71	238.512	379.862	618.955
52	121.696	167.165	232.856	72	246.667	396.057	650.903
53	126.347	174.851	245.499	73	255.067	412.899	684.448
54	131.138	182.845	258.774	74	263.719	430.415	719.670
55	136.072	191.159	272.713	75	272.631	448.631	756.654
56	141.154	199.806	287.348	76	281.810	467.577	795.486
57	146.388	208.798	302.716	77	291.264	487.280	836.261
58	151.780	218.150	318.851	78	301.002	507.771	879.074
59	157.333	227.876	335.794	79	311.032	529.082	924.027
60	163.053	237.991	353.584	80	321.363	551.245	971.229
61	168.945	248.510	372.263	81	332.004	574.295	1020.790
62	175.013	259.451	391.876	82	342.964	598.267	1072.830
63	181.264	270.829	412.470	83	354.253	623.197	1127.471
64	187.702	282.662	434.093	84	365.880	649.125	1184.845
65	194.333	294.968	456.789	85	377.857	676.090	1245.087
66	201.163	307.767	480.638	86	390.193	704.134	1308.341
67	208.198	321.078	505.670	87	402.898	733.299	1374.758
68	215.444	334.921	531.953	88	415.985	763.631	1444.496
69	222.907	349.318	559.551	89	429.465	795.176	1517.721
70	230.594	364.290	588.529	90	443.349	827.983	1594.607

Theſe three Tables are continuations of Tab. III. IV. and V. of compound intereſt, and may be neceſſary, beſides other purpoſes, in the ſolution of queſtions in annuities on lives, when *n*, the complement of life, exceeds 50.

CHAP.

CHAP. VIII.

Of ANNUITIES *on* LIVES.

59. WAS human life certain of continuing to a given period, the ſum of the ſeries $\frac{1}{R}+\frac{1}{R^2}+\frac{1}{R^3}$ $(n)=\frac{1-\frac{1}{R^n}}{r}$ (47.) would expreſs, at a given rate, the value of a perſon's life who had yet n years to live; but ſeeing there is much contingency in the caſe, as we find by daily experience, Tables of the Probabilities of Life, called alſo Tables of Obſervations, have been conſtructed from the bills of mortality; from whence the value of life, at a given age and rate of intereſt, might be computed. Of theſe take the following Table, conſtructed by Dr Halley from the bills of mortality at Breſlaw, as an example.

TABLE

TABLE I. Shewing the Probabilities of Life at Breslaw, Constructed by Dr Halley from the Bills of Mortality in that City.

Age.	Living.	Dead.	Age.	Living.	Dead.	Age.	Living.	Dead.
0	1238	238	31	523	8	62	222	10
1	1000	145	32	515	8	63	212	10
2	855	57	33	507	8	64	202	10
3	798	38	34	499	9	65	192	10
4	760	28	35	490	9	66	182	10
5	732	22	36	481	9	67	172	10
6	710	18	37	472	9	68	162	10
7	692	12	38	463	9	69	152	10
8	630	10	39	454	9	70	142	11
9	670	9	40	445	9	71	131	11
10	661	8	41	436	9	72	120	11
11	653	7	42	427	10	73	109	11
12	646	6	43	417	10	74	98	10
13	640	6	44	407	10	75	88	10
14	634	6	45	397	10	76	78	10
15	628	6	46	387	10	77	68	10
16	622	6	47	377	10	78	58	9
17	616	6	48	367	10	79	49	8
18	610	6	49	357	11	80	41	7
19	604	6	50	346	11	81	34	6
20	598	6	51	335	11	82	28	5
21	592	6	52	324	11	83	23	4
22	586	7	53	313	11	84	19	4
23	579	6	54	302	10	85	15	4
24	573	6	55	292	10	86	11	3
25	567	7	56	282	10	87	8	3
26	560	7	57	272	10	88	5	2
27	553	7	58	262	10	89	3	2
28	546	7	59	252	10	90	1	1
29	539	8	60	242	10	91	0	0
30	531	8	61	232	10			

Such a table as this of the probabilities of life, is formed on the principle, "that the number "dying annually, after every particular age, is "equal to the number living at that age," and, of

of conſequence, is poſſeſſed of the following properties:

1ſt, The ſeveral numbers in column 2. or 3. from infancy to the utmoſt extent of life, will repreſent the perſons living, or who die yearly, at their correſponding ages.

2d, The ſum of all the perſons alive or dead in column 2. or 3. will expreſs the number of inhabitants, or the number of perſons who die yearly, and theſe laſt, it is manifeſt, will be equal to 1238 who are born in any one year; alſo the ſum of the perſons dying yearly, from any one age and upwards, will be equal to the number living at that age.

3d, As the 1000 children, 1 year old, were not all born at the beginning of the year, but ſucceſſively from day to day, they, as well as the perſons living at any ſucceeding age, are repreſented in the table half a year older, at a medium, than they are in fact; of conſequence, in queſtions reſpecting the expectation of a ſingle life, (See Art. 4.) half a year ought to be deducted from the anſwers given by the Table. Hence may be derived the following Articles:

Art. 1. To find the chance or probability which a perſon of a given age hath of living a propoſed number of years. This is diſcovered by the proportion which the living, at the propoſed age, have to thoſe who have died ſince the given one. Thus, Suppoſing a perſon of 30 years of age, then his chance of living is to that of dying,

As 523 : 8 = 65 : 1, in 1 year.
and 472 : 59 = 8 : 1, in 7 years.

Hence,

Hence, to find how long a perſon of a given age hath the chance of living, take half the number of perſons living at the given age, and oppoſite to it in col. 1ſt, you have the years required. Thus, $\frac{531}{2}=265$, which is found in the table between 57 and 58 years; ſo that a perſon aged 30 hath an equal chance of living between 27 and 28 years longer.

Art. 2. To find the probability of life's continuing, or of its failing in 1, 2, 3, &c. years, in a perſon of a given age, when the certainty of the preſent moment, or of death, is expreſſed by unity. Here, ſuppoſing a perſon of 30 years of age, out of 531 chances, he hath 523 of living one year, and eight chances of dying in that time. But by the doctrine of chances, the probability for the happening of an event is expreſſed by a fraction, whoſe numerator is the number of chances for its happening, and denominator, the ſum of the chances both of its happening and failing. Hence,

$$\left.\begin{matrix}\frac{523}{531}\\ \frac{8}{531}\end{matrix}\right\} = \text{Probability that ſuch a perſon} \begin{cases}\text{will live 1 year.}\\ \text{will die in that time.}\end{cases}$$

But the ſum of theſe fractions is equal to unity; therefore, having one, we can find the other by ſubtraction. Hence, $\frac{523}{531}$, $\frac{515}{531}$, $\frac{507}{531}$, will expreſs the probabilities that ſuch a perſon will live 1, 2, 3 years longer; and $\frac{8}{531}$, $\frac{16}{531}$, $\frac{24}{531}$, thoſe of his dying in that time.

Art. 3d. Out of a certain number of perſons of a given age, to find how many of them have a chance of dying yearly.

As the tabular number, at the given age, is to thoſe who die out of them, ſo is the number of perſons alive, to thoſe who die yearly.

Ex. Out of 135 perſons, whoſe age, at a medium, is 55 years, it is required, to find how many have a chance of dying yearly, till the laſt is extinct at the age of 91 years.

292 : 10 : : 135 : 4.623 who die the 1ſt year;
and 282 : 10 : : 130.377 : 4.623 2d year, &c.

Or rather thus: As theſe perſons are in life from 30 years of age and upwards, nearly in the ſame proportion as the tabular numbers from the ſame age; hence,

15031 : 531 : : 135 : 4.77 who die the 1ſt year;
and 14500 : 523 : : 130.23 : 4.70 2d year, &c.

Art. 4th. To find the expectation of life, that is, the number of years which mankind enjoy, taken one with another, either from birth or any age propoſed.

Divide the number of perſons alive at the given age and upwards, by the number alive at the given age, the quotient, minus .5, gives the anſwer. Thus,

$\frac{35173}{1238}$ (minus .5) = 28, the expectation of an infant.

Or thus: The expectation of life being equal to the ſum of the probabilities that a perſon ſhall live from the preſent inſtant, 1, 2, 3, &c. years; the expectation

expectation of life in a perſon, for inſtance, of 30 years of age, is expreſſed by the ſum of the fractions, viz.

$\frac{531}{531}+\frac{523}{531}+\frac{515}{531}$, &c. carried on to the utmoſt extent of life, (minus .5) = 27.87.

Art. 5th. To calculate the value of L. 1 annuity upon a life of a given age, and at a given rate of intereſt.

By Art. 2d. the probabilities of the continuance of life in a perſon, for inſtance, of 30 years of age, for 1, 2, 3, &c. years, are expreſſed by a ſeries of fractions, viz.

$\frac{523}{531}$, $\frac{515}{531}$, $\frac{507}{531}$, &c. But independent of chance, the terms of the ſeries, viz.

$\frac{1}{R}$ $\frac{1}{R^2}$ $\frac{1}{R^3}$. . . . $\frac{1}{R^n}$, ſhew the preſent value of L. 1 due at the end of 1, 2, 3, ... n, years. Therefore, combining theſe two ſeries together,

$\frac{523}{531R}+\frac{515}{531R^2}+\frac{507}{531R^3}$, &c. carried on to the utmoſt extent of life, will expreſs the value of L. 1 annuity, upon the life of a perſon aged 30 years, at a given rate of intereſt. But as the calculation of 60 terms of this ſeries muſt prove a work of labour, let $N = 1.44$ denote at 4 per cent. the value of a life aged 86, which is eaſily found by the above method, and $\frac{a}{b}=\frac{11}{15}$, the probability that a life, aged 85, will ſurvive 1 year; then,

$\frac{a}{bR} \times \overline{1+N} = 1.72$, is the value of a life aged 85 years. Proceeding thus, ſtep by ſtep, the values

of lives, by any table of obſervations, may be found.

Art. 6th. To find the amount of the population in any city or county in the kingdom, from their annual births or burials.

If the annual average of births and burials be equal, either of them multiplied by the expectation of an infant's life taken from a table adapted to that diſtrict, will produce the amount of the population thereof. If this annual average be unequal, which is moſt likely; in this caſe, as the population is as much increaſed by the ſuperior number of births, as it is diminiſhed by the comparative defect in the number of burials; therefore, the half of their ſum, multiplied by an infant's expectation of life, call it 40, (ſee Table VIII.) will produce the number of inhabitants.

Ex. By the collector's accounts of the duty upon baptiſms, &c. within the bounds of the ſynod of Fife, anno 1790,

The annual average of baptiſms = 2484
of burials = 1536

Half their ſum = 2010
Multiplied by 40

Produces 80400, the number of inhabitants within the bounds of the ſynod of Fife.

60. *Of the Hypotheſis of Equal Decrements.*

Mr de Moivre hath invented an hypotheſis, whereby the values of annuities on lives may be determined

determined nearly as accurately as if they had been deduced year by year from a table of obſervations; namely, "That the probabilities of "life, after a certain age, decreaſe in arithmeti- "cal progreſſion." Thus, ſuppoſing, for inſtance, 36 perſons, each of the age of 50, if, after one year expired, there remained but 35, after two 34, after three 33, &c. it is evident that ſuch lives would be extinct in 36 years; and that the probabilities of living 1, 2, 3, &c. years from the age of 50, would be expreſſed by the fractions,

$\frac{35}{36}$, $\frac{34}{36}$, $\frac{33}{36}$, &c, which decreaſe in arithmetical progreſſion: And ſuppoſing further, that the age of 86 is the utmoſt probable extent of human life; the complement of life of a given age will be the number of years which that age wants of 86. Thus, 36 will be the complement of a life aged 50 years.

Admitting, then, that the decrements of life, after 10 years of age, are nearly equal; the following Problems will more eaſily be reſolved, than by a table of obſervations.

PROBLEM I.

61. Suppoſing the decrements of life to be equal, it is required, to find the value of L. 1 annuity upon a life of a given age, and at a given rate of compound intereſt.

Let n repreſent the complement of life.

$p=\frac{1}{R^n}$ = the value of L. 1, due at the end of n years.

$P=\frac{1-p}{r}=$ of L. 1 annuity for n years.

$N=$

N=the value of L. 1 annuity upon a life of a given age. Now, ſince the probabilities of life decreaſe in arithmetical progreſſion, they may be repreſented by the terms of the following ſeries, viz.

$\frac{n-1}{n}, \frac{n-2}{n}, \frac{n-3}{n} \ldots . \frac{n-n}{n}$, which expreſſions may alſo be conſidered as the values of each payment of the annuity in the parts of L. 1. But the terms of the ſeries, $\frac{1}{R}, \frac{1}{R^2}, \frac{1}{R^3} \ldots . \frac{1}{R^n}$, ſhew the preſent values of L. 1, due at the end of 1, 2, 3, ... n years; therefore combining theſe two ſeries together,

$\frac{n-1}{nR}+\frac{n-2}{nR^2}+\frac{n-3}{nR^3}(n)=N$, will expreſs the value of an annuity upon a life, whoſe complement is n, at a given rate of intereſt.

Which ſeries may be divided into two other ſeries, viz.

$\frac{n}{n}\times\overline{\frac{1}{R}+\frac{1}{R^2}+\frac{1}{R^3}}(n)=\frac{1}{1}\times\frac{1-p}{r}$; (47) and $-\frac{1}{n}\times\frac{1}{R}+\frac{2}{R^2}+\frac{3}{R^3}(n)=\frac{1}{n}\times\overline{\frac{RP}{r}-\frac{np}{r}}$ *.

Hence,

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* Sought, the value of n terms of the ſeries, $\frac{1}{R}+\frac{2}{R^2}+\frac{3}{R^3}$, &c.

If the terms of the ſeries $\frac{1}{R}+\frac{2}{R^2}+\frac{3}{R^3}$, &c. be multiplied by $\overline{R-1}^2=R^2+2R+1$

The product $=R+0+0$. Hence, $\frac{R}{\overline{R-1}^2}=\frac{R}{r^2}$ is the value of the ſeries indefinitely continued.

Now

Hence, $\frac{1-p}{r}-\frac{RP}{nr}+\frac{p}{r}=N$; but $\frac{1-p}{r}+\frac{p}{r}=\frac{1}{r}$.

Hence, $\frac{1}{r}-\frac{RP}{nr}=\frac{1-\frac{R}{n}P.}{r}$ $\Bigg\}$ $=N$, the value fought.

Or, $P-\frac{RP-np}{nr}$ $\Bigg\}$

Ex. It is required, to find the preſent value of L. 1 annuity upon the life of a perſon aged 36 years, reckoning intereſt at 4 per cent. Here, $n=50$, $P=21.482$, $R=1.04$, and $r=.04$; hence,

$$\frac{1-\frac{R}{n}P}{r}=\frac{.5532}{.04}=13.83,$$ the value fought.

PROBLEM II.

Suppoſing the decrements of life to be equal, it is required to find the preſent value of L. 1 an-

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Now the terms, which follow the n term, in the given ſeries,

are $\frac{n+1}{R^{n+1}}+\frac{n+2}{R^{n+2}}+\frac{n+3}{R^{n+3}}$, &c. *ad infinitum.*

Or, $\frac{1}{R^n}\times\overline{\frac{n+1}{R}+\frac{n+2}{R^2}+\frac{n+3}{R^3}}$, &c. *ad infinitum.*

Which ſeries may be divided into two other ſeries, viz.

$\frac{1}{R^n}\times\overline{\frac{n}{R}+\frac{n}{R^2}+\frac{n}{R^3}}$, &c. $=\frac{n}{R^n}\times\frac{1}{r}$ (40) $=\frac{np}{r}$; and,

$\frac{1}{R^n}\times\frac{1}{R}+\frac{2}{R^2}+\frac{3}{R^3}$, &c. $=\frac{1}{R^n}\times\frac{R}{r^2}=\frac{Rp}{r^2}$, by the above. Hence, if the ſum of theſe two ſeries be ſubtracted from the former, there will remain

$\frac{R}{r^2}-\frac{Rp}{r^2}-\frac{np}{r}$, or, $\frac{\overline{1-p}\times R}{r^2}-\frac{np}{r}=\frac{RP}{r}-\frac{np}{r}=$ the value of the ſeries $\frac{1}{R}+\frac{2}{R^2}+\frac{3}{R^3}$, (n).

nuity,

nuity, to continue during the joint lives of two perſons, whoſe reſpective complements of life are n and m, (m being the greater number) at a given rate of compound intereſt.

62. Since the probability of the continuance of the life, whoſe complement is n, for one year, is $\frac{n-1}{n}$, and that of the life whoſe complement is m, $\frac{m-1}{m}$; and ſince theſe two events are independent of each other, it follows, that the terms of the ſeries, $\frac{n-1}{n} \times \frac{m-1}{m}$, $\frac{n-2}{n} \times \frac{m-2}{m}$, $\frac{n-3}{n} \times \frac{m-3}{m}$, &c. will reſpectively repreſent the probabilities of their continuing together, 1, 2, 3, &c. years; which probabilities may be conſidered as the values of the ſeveral payments of the annuity, in the parts of L. 1, which will become due at the end of 1, 2, 3, &c. years; and the value of L. 1 annuity upon the two joint lives, will be expreſſed by the following ſeries.

$$\frac{\overline{n-1} \times \overline{m-1}}{nmR} + \frac{\overline{n-2} \times \overline{m-2}}{nmR^2} + \frac{\overline{n-3} \times \overline{m-3}}{nmR^3} \text{ &c.}$$

of which ſeries n terms only will be uſeful, n being the complement of the oldeſt life. Now if the numerators of the fractions in the above ſeries be expanded by multiplication, they will appear in this form.

$$\overline{n-1} \times \overline{m-1} = nm - \overline{m+n} \times 1 + 1.$$
$$\overline{n-2} \times \overline{m-2} = nm - \overline{m+n} \times 2 + 4.$$
$$\overline{n-3} \times \overline{m-3} = nm - \overline{m+n} \times 3 + 9, \text{ &c.}$$

Hence, the above ſeries may be divided into three other ſeries,

viz.

viz. $\frac{nm}{nm} \times \overline{\frac{1}{R}+\frac{1}{R^2}+\frac{1}{R^3}}, (n) = \frac{nm}{nm} \times \frac{1-p}{r}$ (47.)

$$-\frac{m+n}{nm} \times \overline{\frac{1}{R}+\frac{2}{R^2}+\frac{3}{R^3}}, (n) = \frac{m+n}{nm} \times \frac{\overline{1-pR} \times -np}{r^2}, \frac{}{r}$$

(By note, Prob. 1.)

$$+\frac{1}{nm} \times \overline{\frac{1}{R}+\frac{4}{R^2}+\frac{9}{R^3}}(n) = \frac{1}{nm} \times \frac{\overline{1-p} \times \overline{R+1} \times R - 2nRp}{r^3}, \frac{n^2p}{r} *$$

The sum of which three series, being properly ranged, may stand thus,

$$\frac{nm}{nm} \times \frac{1-p}{r}.$$

$$+\frac{m+n}{nm} \times \frac{np}{r} - \frac{m+n}{nm} \times \frac{\overline{1+p} \times R}{r^2}.$$

$$-\frac{1}{nm} \times \frac{nnp}{r} - \frac{1}{nm} \times \frac{2npR}{r^2} + \frac{1}{nm} \times \frac{\overline{1-p} \times \overline{R+1} \times R}{r^3}.$$

Their sum $= \frac{1}{r} - \frac{\overline{m+n} \times R - m - n \times Rp}{nm \times r^2} + \frac{\overline{1-p} \times \overline{R+1} \times R}{nm \times r^3}$

= the value of the annuity.

Ex.

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* If the series $\frac{1}{R}+\frac{4}{R^2}+\frac{9}{R^3}$ &c. be multiplied by $\overline{R-1}^3 = R^3+3R^2+3R-1$ the product $= R^2+R+0 = \overline{R+1} \times R$; therefore, $\frac{\overline{R+1} \times R}{r^3}$ = the value of the above series indefinitely continued.

Now the terms which follow the n term in the series are,

$$\frac{\overline{n+1}^2}{R^{n+1}} + \frac{\overline{n+2}^2}{R^{n+2}} + \frac{\overline{n+3}^2}{R^{n+3}}, \text{ \&c.}$$

$$\text{Or, } \frac{nn+2n+1}{R^{n+1}} + \frac{nn+4n+4}{R^{n+2}} + \frac{nn+6n+9}{R^{n+3}}, \text{ \&c.}$$

Which series may be divided into three others, viz.

$$\frac{nn}{R^n} \times \overline{\frac{1}{R}+\frac{1}{R^2}+\frac{1}{R^3}}, \text{ \&c.} = \frac{nn}{R^n} \times \frac{1}{r} \quad (40.)$$

$$\frac{2n}{R^n} \times \overline{\frac{1}{R}+\frac{2}{R^2}+\frac{3}{R^3}}, \text{ \&c.} = \frac{2n}{R^n} \times \frac{R}{r^2} \text{ (last note)}$$

$$\frac{1}{R^n} \times \overline{\frac{1}{R}+\frac{4}{R^2}+\frac{9}{R^3}}, \text{ \&c.} = \frac{1}{R^n} \times \frac{\overline{R+1} \times R}{r^3}, \text{ as above.}$$

Ex. It is required to find the preſent value of L. 1 annuity, to continue during the joint lives of two perſons, whoſe reſpective ages are 54 and 43 years, reckoning intereſt at 4 per cent. Here $n=32$, $m=43$, $p=.285$, $R=1.04$ and $r=.04$. Hence,

$$\frac{1}{r} \quad - \quad - \quad - \quad = 25.000$$

$$+\frac{\overline{1-p}\times\overline{R+1}\times R}{nm\times r^3} = \frac{1.5168}{.088} = \frac{17.224}{42.224}$$

$$-\frac{\overline{m+n}\times R-\overline{m-n}\times Rp}{nm\times r^2} = \frac{74.730}{2.2} = \frac{33.947}{8.277}$$

The value of the annuity $=$ 8.277

When the ſingle lives are equal, $m+n=2n$, $\overline{m-n}\times Rp=o$, $nm=n^2$, and the negative quantity $=-\frac{2R}{nr^2}$

Ex. Sought, the value of L. 1 annuity to continue during the joint continuance of two lives, each aged 48 years, reckoning intereſt at 4 per cent. Here,

$$\frac{1}{r}+\frac{\overline{1-p}\times\overline{R+1}\times R}{n^2\times r^3} = 42.785.$$

$$-\frac{2R}{nr^2} = 34.215.$$

The value ſought $=$ 8.570.

In which three ſeries, making $p=\frac{1}{R^n}$, and ſubtracting their ſum from the above found value of the whole ſeries, there will remain $\frac{1-p\times\overline{R+1}\times R}{r^2}-\frac{2npR}{r^2}-\frac{npp}{r}=$ the value of n terms thereof.

PROB-

PROBLEM III.

To find the preſent value of L. 1 annuity, either upon the joint continuance, or during the longeſt liver, of any number of lives, at a given rate of intereſt.

63. 1ſt, As the value of joint lives enters an ingredient into almoſt every queſtion reſpecting life-annuities, this value, at 4 per cent. may be diſcovered by the following conciſe and accurate method. Let n, m, p, and t, repreſent the complements of life; c, the arithmetical mean complement of all the lives, excepting the oldeſt, whoſe value, at 4 per cent. is N; and $N - \frac{a}{b}N$, the value of the joint lives. Then,

In the caſe of 2, 3, 4, equal joint lives, with a ſmall correction, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, = the *ratio* of the value of the joint lives to that of the ſingle one; hence,

$\frac{a}{b}$, with a ſmall correction, $= \frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2} = 1$ — the *ratio* of the value of the joint lives to that of the ſingle one.

Again, in the caſe of 2 / —— of 3 or 4 } equal joint lives, $n = \begin{cases} m. \\ c. \end{cases}$

But when the lives are unequal, $\frac{n}{m}$ or $\frac{n}{c}$ will correct the irregularity ariſing from the inequality of the lives. Hence the following

TABLE.

Lives.	Corrections at 4 per cent.								
	$n=76$	66	56	51	46	36	26	16	6
2	$N-\frac{n-12}{4m}N$	−7	−3	−1	+ 1	+ 3	+ 4	+ 5	+3
3	$N-\frac{n+8}{3c}N$	+9	+10	+10	+10	+10	+ 9	+ 6	+3
4	$N-\frac{n-6}{2e}N$	−3	−1	0	+ 1	+ 2	+ 3	+ 2	+1

Ex. Sought, the value of two joint lives, whoſe reſpective ages are 46 and 30 years, at 4 per cent. Here $n=40$, $m=56$, and $N-\frac{n+2.2}{4m}N=12.135-2.286=9.849$, the value.

Or, the preſent value of joint lives, at a given rate of intereſt, may be found univerſally. Thus:

Let a repreſent the expectation of their joint continuance, (ſee ſupplement to problem 18.); b, that of the oldeſt; and A, the number of years, by Table V. of annuities certain, correſponding to N, the value of the oldeſt life *. Then, ſeeing $b:a::A:\frac{a}{b}A$, the value by the ſaid table, anſwering to $\frac{a}{b}A$ years certain †, (or in the caſe of two joint lives, the value correſponding to $A-\frac{n}{3m}A$ years certain), will be that of their joint continuance.

Ex. Let there be five equal lives, each aged 32

* Let the age be 32, whoſe value, at 4 per cent is 14.411; the next leſs, to which, by Table V. is 14.029, anſwering to 21 years; their difference is .382, and the increaſe of value for one year is .422; hence, $\frac{.382}{.422}=.905$, to be annexed to 21, making, in all, 21.905 years.

† Let the number of years certain be 32.86, the value of 32 years, at 4 per cent. is 17.873, and the increaſe of value for one year is .274; hence $.274\times.86=.235$ to be added to 17.873, making their ſum $=18.108$, the value ſought.

years;

years; it is required to find the value of their joint continuance, at 4 per cent. Here, $a=9$, $b=27$, and $A=21.9$; hence, $\frac{a}{b}A=7.3$ years, whoſe value, viz. 6.46, is that of the joint continuance of theſe five lives.

Or, the preſent value of L. 1 annuity upon the joint continuance of any number of lives, at a given rate, may be found univerſally, thus:

2d, Let N=expectation of the oldeſt life, whoſe complement is n; x=expectation of the joint continuance of all the lives; and A=number of years certain, correſponding to the value of the oldeſt life. Then,

Seeing $N : x :: A : \frac{x}{N}A$, the value correſponding to $\frac{x}{N}A$ years certain, (or in the caſe of two lives, the value anſwering to $A-\frac{n}{3m}A$ years), will be that of their joint continuance.

Ex. Let there be three lives, whoſe reſpective ages are 46, 36, and 26 years; it is required to find the value of their joint continuance, reckoning intereſt at 5 per cent. Here,

$N=20$, $x=12$, and $A=16.35$; hence,

$\frac{12}{20}\times 16.35=9.81$ years, whoſe value, viz. 7.6, is that of the joint continuance of theſe three lives.

In the caſe of m equal joint lives, $\frac{x}{N}=\frac{2}{m+1}$.

23d, To find the value of L. 1 annuity upon the longeſt liver of any number of lives, at a given rate.

Let n=complement of the mean life, whoſe expectation is N; X or x=expectation of the longeſt liver, or of joint continuance of all the lives; and $d=N-$the number of years certain, correſponding

ing to the value of the mean life, at a given rate. Then,

Seeing $n-N=N : n-X=x :: d : \frac{x}{N} \times d$, the value corresponding to $X-\frac{x}{N} \times d$ years certain, will be that of the longeſt liver.

Ex. Let there be four equal lives, whoſe common age is 36 years; ſought, the value of the longeſt liver, reckoning intereſt at 4 per cent. Here,

$n=50$, $N=25$, $X=40$, $x=10$, and $d=4.46$. Hence,

$40-\frac{4.46}{25} \times 10=38.2$ years, whoſe value, viz. 19.4, is that of the longeſt liver of theſe 4 lives.

In the caſe of m equal lives, $X-\frac{2d}{m+1}=$ the number of years certain, correſponding to the value of the longeſt liver.

Theſe two methods of finding the value of the joint continuance, &c. of any number of lives, are more accurate than Mr de Moivre's Theorms for the ſame purpoſe; and more conciſe than thoſe of Mr Dodſon in his Mathematical Repoſitory, vol. 2. and may be applied in finding the value of lives, by a table of obſervations, as well as by the hypotheſis.

Ex. It is required to find, by Table VIII. and IX. the value of the joint continuance of two lives, whoſe reſpective ages are 30 and 25 years, at 4 per cent. Here,

$n=65.08$, $m=69.98$, and $A=27.244$; hence,

$A-\frac{n}{3m}A=18.8$ years, whoſe value, viz. 13, is that of the joint continuance of theſe two lives.

TABLE

TABLE II. Shewing the preſent Values of L. 1 Annuity on a ſingle Life, according to Mr de Moivre's Hypotheſis of equal Decrements.

Age.	3 per cent.	4 per cent.	5 per cent.	Age.	3 per cent.	4 per cent.	5 per cent.
9–10	19.868	16.882	14.607	48	13.012	11.748	10.679
8–11	19.736	16.791	14.544	49	12.764	11.548	10.515
7–12	19 604	16.698	14.480	50	12.511	11.344	10.348
13	19.469	16.604	14.412	51	12.255	11.135	10.176
6–14	19.331	16.508	14.342	52	11.994	10.921	9.999
15	19.192	16.410	14.271	53	11 729	10.702	9.817
16	19.050	16.311	14.197	54	11.457	10.478	9.630
5–17	18.905	16.209	14.123	55	11.183	10.248	9.437
18	18.759	16.105	14.047	56	10.902	10.014	9.239
19	18.610	15.999	13.970	57	10.616	9.773	9.036
4–20	18.458	15.891	13.891	58	10.325	9.527	8.826
21	18.305	15.781	13.810	59	10.029	9.275	8.611
22	18.148	15.669	13.727	60	9.727	9.017	8.389
23	17.990	15.554	13.642	61	9.419	8.753	8.161
3–24	17.827	15.437	13.555	62	9.107	8.482	7.926
25	17.664	15.318	13.466	63	8.787	8.205	7.684
26	17.497	15.197	13.375	64	8.462	7.921	7.435
27	17.327	15.073	13.282	65	8.132	7.631	7.179
28	17.154	14.946	13.186	66	7.794	7.333	6.915
29	16.979	14.816	13.088	67	7.450	7.027	6.643
30	16.800	14.684	12.988	68	7.099	6.714	6.362
2–31	16.620	14.549	12.885	69	6.743	6.394	6.073
32	16.436	14.411	12.780	70	6.378	6.065	5.775
33	16.248	14.270	12.673	71	6.008	5.728	5.468
34	16.057	14.126	12.562	72	5.631	5.383	5.152
35	15.867	13.979	12.449	73	5.246	5.029	4.826
36	15.666	13.829	12.333	74	4.854	4.666	4.489
37	15.465	13.676	12.214	75	4.453	4.293	4.143
38	15.260	13.519	12.091	76	4.046	3.912	3.784
39	15.053	13.359	11.966	77	3.632	3.520	3.415
40	14.841	13.196	11.837	78	3.207	3.111	3.034
41	14.626	13.028	11.705	79	2.776	2.707	2.641
42	14.407	12.858	11.570	80	2.334	2.284	2.235
43	14.185	12.683	11.431	81	1.886	1.850	1.816
44	13.958	12.504	11.288	82	1.429	1.406	1.384
45	13.728	12.322	11.142	83	0.961	0.950	0.937
46	13.493	12.135	10 992	84	0.484	0.481	0.476
47	13.254	11.944	10.837	85	0.000	0.000	0.000

Table II. is conſtructed by the theorem $\frac{1-\frac{R}{n}p}{r}$, which expreſſes the value of a ſingle life, whoſe complement is n.

Or thus, let the ſum of the following ſeries, viz.

$\frac{1}{R}$.

$\frac{1}{R}+\frac{1}{R^2}$

$\frac{1}{R}+\frac{1}{R^2}+\frac{1}{R^3}$

$\frac{1}{R}+\frac{1}{R^2}+\frac{1}{R^3}+\frac{1}{R^4}$

&c.

Their ſum $=\overline{n-1}\frac{1}{R}+\overline{n-2}\frac{1}{R^2}+\overline{n-3}\frac{1}{R^3}\cdots+\frac{1}{R^{n-1}}$ be divided by n $=$ the preſent worth of L. 1 annuity upon a ſingle life, whoſe complement is n, at a given rate of intereſt, and is the ſame with the ſeries of equal decrements, Problem I. wanting the laſt term, viz. $\frac{n-n}{n}R^n=0$. Hence the ſum of $n-1$ terms, in Table V. of annuities certain, divided by n, will give the value of a life whoſe complement is n.

Ex. Let the age be 70, whoſe value is ſought, at 4 per cent. Here, $n=16$, and

15 terms of Table V. $=97.04$

divided by 16 $=6.065$ the value ſought.

This affords an accurate and eaſy method of finding the value of ſingle or conjoint lives, &c. at a given rate, not only by the hypotheſis, but alſo by

by a table of obſervations, when the expectation of life is known.

Ex. By Table VIII. 22.42 = expectation of the joint continuance of two lives, whoſe reſpective ages are 30 and 25; ſought, their value at 4 per cent. Here, $n = 44.84$ and 43.84 terms of Table V. $= 582.96$ divided by $44.84 = 13$, { the value of the joint continuance of theſe two lives.

Again, let N denote the value of a ſingle life of a given age, $\frac{a}{b} = \frac{n}{n+1}$ the probability that a life will continue for one year preceding that correſponding to N; then,

$\frac{a}{bR} \times \overline{1+N}$ = the value of a ſingle life one year younger than that correſponding to N.

As the difference of the terms in Table V. of annuities certain doth conſtantly decreaſe, and ſeeing $n-1$ terms are divided by n, in finding the value of a life whoſe complement is n; hence, the preſent value of a ſingle life will always be leſs than that of its expectation, conſidered as ſo many years certain.

Whereas the amount of an annuity on a given life (ſee Table IV.) is greater than that during its expectation; yet ſuch is the accuracy of calculation, that the ſum of $N \times n$, improven during life, will exactly balance the n number of life-annuities, each of whoſe values is N, whether they be paid regularly as they become due, or are diſcharged by ſingle payments at the cloſe of each life.

TABLE III. Shewing the Value of L. 1 Annuity on the Joint Continuance of two Lives, ſuppoſing the Decrements of Life to be equal.

Age	Age.	3 pr cent.	4 pr cent.	5 pr cent.
	10	15.206	13.342	11.855
	15	14.878	13.093	11.661
	20	14.503	12.808	11.430
	25	14.074	12.480	11.182
	30	13.585	12.102	10.884
	35	13.025	11.665	10.537
10	40	12.381	11.156	10.128
	45	11.644	10.564	9.646
	50	10.796	9.871	9.074
	55	9.822	9.059	8.391
	60	8.704	8.105	7.572
	65	7.417	6.980	6.585
	70	5.936	5.652	5.391
	15	14.574	12.860	11.478
	20	14.225	12.593	11.266
	25	13.822	12.281	11.022
	30	13.359	11.921	10.736
	35	12.824	11.501	10.402
15	40	12.207	11.013	10.008
	45	11.496	10.440	9.541
	50	10.675	9.767	8.985
	55	9.727	8.975	8.318
	60	8.632	8.041	7.515
	65	7.377	6.934	6.544
	70	5.932	5.623	5.364
	20	13.904	12.341	11.067
	25	13.531	12.051	10.840
	30	13.098	11.711	10.565
	35	12.594	11.314	10.278
	40	12.008	10.847	9.870
20	45	11.325	10.297	9.420
	50	10.536	9.648	8.880
	55	9.617	8.879	8.233
	60	8.549	7.967	7.448
	65	7.308	6.882	6.495
	70	5.868	5.590	5.338
	25	13.192	11.786	10.621
	30	12.792	11.468	10.367
	35	12.333	11.095	10.067
	40	11.776	10.655	9.708
25	45	11.130	10.131	9.278
	50	10.374	9.509	8.761
	55	9.488	8.766	8.134
	60	8.452	7.880	7.371
	65	7.241	6.826	6.440
	70	5.826	5.551	5.294

Age.	Age.	3 pr cent.	4 pr cent.	5 pr cent.
	30	12.434	11.182	10.133
	35	12.010	10.838	9.854
	40	11.502	10.428	9.514
	45	10.898	9.936	9.112
30	50	10.183	9.345	8.620
	55	9.338	8.634	8.018
	60	8.338	7.779	7.280
	65	7.161	6.748	6.373
	70	5.777	5.505	5.254
	35	11.632	10.530	9.600
	40	11.175	10.157	9.291
	45	10.622	9.702	8.913
	50	9.955	9.149	8.450
35	55	9.156	8.476	7.879
	60	8.202	7.658	7.172
	65	7.066	6.662	6.294
	70	5.718	5.430	5.203
	40	10.777	9.826	9.024
	45	10.283	9.418	8.671
	50	9.677	8.911	8.244
40	55	8.936	8.288	7.710
	60	8.038	7.510	7.039
	65	6.951	6.536	6.198
	70	5.646	5.383	5.141
	45	9.863	9.063	8.370
	50	9.331	8.619	7.987
	55	8.662	8.044	7.500
45	60	7.831	7.332	6.875
	65	6.807	6.425	6.080
	70	5.556	5.300	5.063
	50	8.892	8.235	7.660
	55	8.312	7.738	7.230
50	60	7.568	7.091	6.664
	65	6.623	6.258	5.926
	70	5.442	5.183	4.964
	55	7.849	7.332	6.873
	60	7.220	6.781	6.386
	65	6.379	6.036	5.724
55	70	5.291	5.053	4.823
	60	6.737	6.351	6.001
60	65	6.043	5.730	5.444
	70	5.081	4.858	4.653
	65	5.547	5.277	5.031
65	70	4.773	4.571	4.385
70	70	4.270	4.104	3.952

In

In the conſtruction of Table III. Dr Price made uſe of the theorem $V-\frac{v+1}{m}\times\overline{m-n-2v-1}\times\overline{\frac{p}{n}+2v}=NM$, expreſſing the value of the joint continuance of two lives, whoſe complements are n and m. Here,

v, p } repreſents { the perpetuity $=\frac{1}{r}$, at a given rate. / the value of L 1. annuity for n years certain.

If only one, or neither of the ages, are to be found in the table, the value of their joint continuance, at a given rate, may be found by the theorem, $N-\frac{n+x}{4m}N$.

Ex. Let there be two lives, whoſe reſpective ages are 38 and 22 years; ſought, the value of their joint continuance, at 4 per cent. Here $N-\frac{n+3}{4m}N=13.519-2.55=10.969$, the value ſought.

Or thus,

1ſt, Let there be two lives, aged 35 and 22 years; ſought, the value of their joint continuance, at 4 per cent.

From the value of 20 by 35 = 11.314
Subtract that of 25 by 35 = 11.095

The remainder $\times\frac{2}{5}=.4\times.219=.088$; which product, ſubtracted from 11.314, leaves 11.226 = the value ſought.

2d, Let there be two lives, aged 38 and 22 years; ſought, the value of their joint continuance, at 4 per cent.

From the value found above = 11.226
Subtract that of 22 by 40 = 10.770

The remainder × $\frac{1}{5}$ = .6 × .456 = .274, which product, ſubtracted from 11.226, leaves 10.952, the value ſought.

TABLE IV. Shewing the amount of L. 1 Annuity foreborn, during a Life of a given Age, at 4 per cent.

Ages.	Amount.	Ages.	Amount.	Ages.	Amount.	Ages.	Amount.
9–10	128 81	29	66.58	48	31.51	67	11.36
8–11	124.54	30	64.20	49	30.15	68	10 59
7–12	120.41	31	61.89	50	28.89	69	9.84
13	116.40	32	59.65	51	27.61	70	9.10
6–14	12.52	33	57.48	52	26 37	71	8.38
15	108.74	34	55.37	53	25.16	72	7.67
16	105.11	35	53.32	54	24.00	73	6.98
5–17	101.66	36	51.34	55	22.85	74	6.31
18	98.13	37	49.41	56	21.75	75	5.65
19	94.81	38	47.53	57	20.67	76	5.02
4–20	91 58	39	45.72	58	19.62	77	4.39
21	88 44	40	43.95	59	18.61	78	3.79
22	85.41	41	42.24	60	17.62	79	3.20
23	82.47	42	40.56	61	16.66	80	2.63
3–24	79.61	43	38.96	62	5.72	81	2.08
25	76 85	44	37.39	63	14 80	82	1.54
26	74.17	45	35.87	64	13.92	83	1.01
27	71 56	46	34 38	65	13.06	84	0.50
28	69 04	47	32.92	66	12 25	85	0.00

In the conſtruction of this table, which hath not been attempted by any author,

Let the ſum of the following ſeries, viz.

$$\begin{array}{l} 1 \\ 1+R \\ 1+R+R^2 \\ 1+R+R^2+R^3 \\ 1+R+R^2+R^3+R \\ \qquad \&c. \end{array}$$

Their ſum $= n-1+\overline{n-2}R+\overline{n-3}R^2 \ldots\ldots + R^{n-2}$ be divided by n $=$ the amount of L. 1 annuity upon a ſingle life, whoſe complement is n, at a given rate of intereſt.

Hence, the ſum of $\overline{n-1}$ terms, in Table IV. of annuities certain, divided by n, will give the amount of L. 1 annuity upon a ſingle life whoſe complement is n.

Ex. Sought, the amount of L. 1 annuity during a life aged 80 years, at 4 per cent. Here $n=6$, and the five firſt terms of Table IV. $= 15.8244$ divided by $6 = 2.64 =$ the amount ſought.

Or thus: Let s denote the amount of L. 1 annuity upon a life of a given age, at a given rate, $\frac{a}{b}$ the probability that a life will continue during one year preceding to that correſponding to s; then $\frac{a}{b} \times \overline{sR+1}$ = the amount of L. 1 annuity upon a ſingle life, one year younger than that correſponding to s; hence the foregoing table.

As the difference of the terms in Table IV. of annuities certain, doth conſtantly increaſe; and although the $n-1$ number of terms is divided by n, in finding the amount of L. 1 annuity upon a life whoſe complement is n, yet in all ages below

low 68, that amount, at 4 per cent. will be greater than that of their expectation, conſidered as ſo many years certain.

By this table, the amount of p, a ſum of money, whoſe intereſt, at 4 per cent. remains unpaid during a ſingle life of a given age, $= 1 + rs \times p$.

TABLE V. Shewing the true Probabilities of Life in London, for all ages, formed by Dr Price, from the Bills of Mortality for 10 Years preceding Anno 1769.

Ages.	Living.	Dead.	Ages.	Living.	Dead.	Ages.	Living.	Dd.	Ages.	Liv.	Dd.
0	1518	486	24	463	8	48	242	9	72	64	6
1	1032	200	25	455	8	49	233	9	73	58	5
2	832	85	26	447	8	50	224	9	74	53	5
3	747	59	27	439	8	51	215	9	75	48	5
4	688	42	28	431	9	52	206	8	76	43	5
5	646	23	29	422	9	53	198	8	77	38	5
6	623	20	30	413	9	54	190	7	78	33	4
7	603	14	31	404	9	55	183	7	79	29	4
8	589	12	32	395	9	56	176	7	80	25	3
9	577	10	33	386	9	57	169	7	81	22	3
10	567	9	34	377	9	58	162	7	82	19	3
11	558	9	35	368	9	59	155	8	83	16	3
12	549	8	36	359	9	60	147	8	84	13	2
13	541	7	37	350	9	61	139	7	85	11	2
14	534	6	38	341	9	62	132	7	86	9	2
15	528	6	39	332	10	63	125	7	87	7	2
16	522	7	40	322	10	64	118	7	88	5	1
17	515	7	41	312	10	65	111	7	89	4	1
18	508	7	42	302	10	66	104	7	90	3	1
19	501	7	43	292	10	67	97	7	91	2	1
20	494	7	44	282	10	68	90	7	92	1	1
21	487	8	45	272	10	69	83	7	93	0	0
22	479	8	46	262	10	70	76	6			
23	471	8	47	252	10	71	70	6			

By

By Table V. the number of births or burials, which are ſuppoſed to be equal, is to that of inhabitants as 1518 : 27935 — 759; that is, as 1 : 17.90; one out of forty who are born in London arrives at 80 years of age, and the expectation of life is as follows:

Ages.	Expectat.	Ages	Expectat.	Ages.	Expectat.	Ages.	Expect.
0	17.90	25	26.67	50	15.90	75	6.33
5	35.23	30	24.12	55	13.93	80	4.98
10	34.92	35	21.77	60	11.71	85	3.32
15	32.32	40	19.51	65	9.71	90	1.50
20	29.37	45	17.64	70	8.77		

TABLE

TABLE VI. Shewing the Value of L. 1 Annuity upon a ſingle Life, according to the true Probabilities of Life in London.

Ages.	3p.cent.	4p.cent.	5p.cent.	Ages	3p.cent.	4p.cent.	5per c.
8— 9	19.13	16.39	14.27	51	11.00	10.07	9.26
7—10	19.05	16.35	14.25	52	10.83	9.93	9.15
11	18.94	16.28	14.21	53	10.60	9.75	8.97
12	18.83	16.21	14.16	54	10.40	9.56	8.82
6—13	18.69	16.10	14.09	55	10.13	9.32	8.61
14	18.50	15.97	13.99	56	9.85	9.08	8.40
5—15	18.27	15.80	13.85	57	9.56	8.83	8.19
16	18.04	15.62	13.71	58	9.28	8.58	7.97
17	17.83	15.46	13.59	59	8.94	8.33	7.75
4—18	17.61	15.30	13.47	60	8.76	8.13	7.58
19	17.39	15.14	13.34	61	8.54	7.94	7.41
20	17.17	14.96	13.20	62	8.26	7.69	7.20
21	16.94	14.79	13.06	63	7.98	7.45	6.98
22	16.74	14.63	12.94	64	7.71	7.21	6.76
23	16.53	14.48	12.82	65	7.44	6.97	6.55
3—24	16.32	14.32	12.69	66	7.18	6.74	6.34
25	16.11	14.15	12.56	67	6.93	6.52	6.14
26	15.89	13.98	12.43	68	6.69	6.31	5.95
27	15.66	13.89	12.29	69	6.48	6.11	5.77
28	15.43	13.62	12.14	70	6.29	5.94	5.62
29	15.24	13.47	12.02	71	6.03	5.71	5.40
2—30	15.03	13.31	11.89	72	5.79	5.50	5.20
31	14.83	13.16	11.77	73	5.58	5.31	5.04
32	14.62	12.99	11.64	74	5.29	5.04	4.79
33	14.41	12.83	11.50	75	5.02	4.79	4.56
34	14.20	12.66	11.37	76	4.77	4.56	4.34
35	13.98	12.49	11.23	77	4.56	4.37	4.16
36	13.76	12.31	11.08	78	4.41	4.23	4.03
37	13.54	12.13	10.94	79	4.17	4.00	3.82
38	13.32	11.95	10.79	80	3.98	3.83	3.65
39	13.12	11.77	10.63	81	3.66	3.54	3.42
40	12.93	11.62	10.51	82	3.37	3.26	3.15
41	12.75	11.47	10.39	83	3.12	3.02	2.93
42	12.56	11.32	10.27	84	2.95	2.87	2.79
43	12.38	11.17	10.15	85	2.60	2.53	2.46
44	12.21	11.03	10.04	86	2.37	2.21	2.16
45	12.04	10.90	9.93	87	2.00	1.96	1.92
46	11.87	10.77	9.82	88	1.89	1.85	1.82
47	11.72	10.64	9.72	89	1.43	1.41	1.38
48	11.57	10.53	9.61	90	0.96	0.95	0.94
49	11.37	10.37	9.50	91	0.48	0.48	0.48
50	11.19	10.22	9.38				

Tables VI. and IX. are conſtructed by the theorem ${}^{a}_{bR} \times \overline{1+N}$, which expreſſes the value of a ſingle life, one year younger than that correſponding to *N*.

TABLE

TABLE VII. Shewing the Value of L. 1 Annuity upon two joint Lives, according to the true Probabilities of Life in London.

Age.	Age.	3p.cent.	4p.cent.	5p.cent.
	10	14.39	12.79	11.29
	15	14.09	12.54	11.10
	20	13.54	12.03	10.79
	25	12.97	11.56	10.44
	30	12.32	11.01	10.02
	35	11.68	10.50	9.60
10	40	10.96	9.91	9.10
	45	10.39	9.41	8.70
	50	9.85	8.93	8.31
	55	9.01	8.26	7.74
	60	7.91	7.32	6.90
	65	6.86	6.37	6.04
	70	5.85	5.46	5.22
	15	13.71	12.22	10.89
	20	13.06	11.79	10.57
	25	12.55	11.34	10.24
	30	11.98	10.83	9.85
	35	11.38	10.32	9.44
15	40	10.73	9.76	8.98
	45	10.16	9.29	8.60
	50	9.60	8.83	8.22
	55	8.84	8.18	7.67
	60	7.81	7.26	6.84
	65	6.83	6.32	6.00
	70	5.81	5.43	5.19
	20	12.76	11.47	10.26
	25	12.27	11.04	9.95
	30	11.76	10.58	9.61
	35	11.19	10.10	9.24
	40	10.58	9.57	8.81
20	45	10.05	9.14	8.47
	50	9.50	8.69	8.10
	55	8.75	8.06	7.56
	60	7.75	7.17	6.76
	65	6.73	6.25	5.93
	70	5.76	5.38	5.15
	25	11.87	10.74	9.66
	30	11.40	10.30	9.36
	35	10.89	9.86	9.03
	40	10.33	9.36	8.63
	45	9.82	8.94	8.29
25	50	9.32	8.53	7.96
	55	8.62	7.94	7.46
	60	7.64	7.07	6.68
	65	6.66	6.18	5.88
	70	5.71	5.32	5.10

Age.	Age	3p.cent.	4p cent.	5p.cent.
	30	10.98	9.97	9.04
	35	10.54	9.58	8.75
	40	10.04	9.13	8.40
	45	9.57	8.74	8.10
30	50	9.11	8.36	7.80
	55	8.44	7.76	7.32
	60	7.52	6.96	6.59
	65	6.58	6.10	5.81
	70	5.65	5.25	5.04
	35	10.14	9.30	8.38
	40	9.70	8.86	8.15
	45	9.28	8.50	7.88
	50	8.88	8.15	7.61
35	55	8.26	7.63	7.18
	60	7.38	6.84	6.48
	65	6.47	6.00	5.72
	70	5.58	5.18	4.97
	40	9.19	8.54	7.84
	45	8.86	8.23	7.61
	50	8.53	7.92	7.38
40	55	7.96	7.42	6.98
	60	7.17	6.69	6.34
	65	6.33	5.89	5.62
	70	5.50	5.09	4.90
	45	8.61	7.94	7.33
	50	8.30	7.67	7.15
	55	7.85	7.23	6.81
45	60	7.05	6.53	6.20
	65	6.24	5.78	5.52
	70	5.41	5.00	4.81
	50	7.96	7.39	6.89
	55	7.53	7.00	6.59
50	60	6.85	6.36	6.04
	65	6.09	5.65	5.40
	70	5.29	4.90	4.72
	55	7.16	6.67	6.23
	60	6.57	6.11	5.80
55	65	5.88	5.46	5.22
	70	5.12	4.75	4.59
	60	6.14	5.72	5.44
60	65	5.56	5.18	4.97
	70	4.36	4.52	4.39
	65	5.22	4.81	4.64
65	70	4.52	4.23	4.11
70	70	4.33	4.10	3.94

TABLE VIII. Shewing the Probabilities of Life, according to the Bills of Mortality in Kettle, for 20 Years, preceding Anno 1790.

Ages	Living.	Dead.	Ages	Living.	Dd.	Ages	Living.	Dd.	Ages.	Living.	Dd.
0	625	20	24	392	6	48	295	5	72	104	8
1	605	40	25	386	6	49	290	6	73	96	8
2	565	36	26	380	6	50	284	7	74	88	8
3	529	20	27	374	6	51	277	7	75	80	8
4	509	16	28	368	5	52	270	7	76	72	8
5	493	12	29	363	5	53	263	7	77	64	8
6	481	6	30	358	4	54	256	7	78	56	7
7	475	4	31	354	4	55	249	7	79	49	6
8	471	4	32	350	4	56	242	7	80	43	5
9	467	4	33	346	4	57	235	8	81	38	4
10	463	4	34	342	4	58	227	8	82	34	3
11	459	4	35	338	4	59	219	8	83	31	3
12	455	4	36	334	4	60	211	9	84	28	3
13	451	5	37	330	4	61	202	9	85	25	3
14	446	5	38	326	3	62	193	9	86	22	3
15	441	5	39	323	3	63	184	9	87	19	3
16	436	5	40	320	3	64	175	9	88	16	3
17	431	5	41	317	3	65	166	9	89	13	3
18	426	5	42	314	3	66	157	9	90	10	3
19	421	5	43	311	3	67	148	9	91	7	2
20	416	6	44	308	3	68	139	9	92	5	2
21	410	6	45	305	3	69	130	9	93	3	1
22	404	6	46	302	3	70	121	8	94	2	1
23	398	6	47	299	4	71	112	8	95	1	1

By Table VIII. the number of births or burials is, to that of inhabitants, as 625 : 25368 — 312.5; that is, as 1 : 40.1; 2 out of 29 arrive at the age of 80 years, and the expectation of life is as follows:

Ages.	Expectat.	Ages	Expectat.	Ages.	Expectat.	Ages.	Expect.
0	40.10	25	34.99	50	18.39	75	7.23
5	45.21	30	32.54	55	15.63	80	6.41
10	43.01	35	29.31	60	12.98	85	4.42
15	40.03	40	25.83	65	10.82	90	2.30
20	37.29	45	21.98	70	8.91	95	0.50

Ex.

Ex. Let there be two lives, whoſe reſpective ages are 40 and 30 years; then

$N = 25.83$, $M = 32.54$ } the double of which { $51.66 = n$. $65.08 = m$.

$N - \frac{n}{3m}N = 19.00 =$ { $\overline{NM}$, expectation of their joint continuance.

$N - \overline{NM} = 6.83 =$ } the expectation of ſurvivorſhip on the part of { the oldeſt.

$M - \overline{NM} = 13.54 =$ } the expectation of ſurvivorſhip on the part of { the youngeſt.

$N + M - {}_2\overline{NM} = 20.37 =$ } the expectation of ſurvivorſhip on the part of { both. And

$N + M - \overline{NM} = M + \frac{n}{3m}N = 39.37$, { the expectation of the longeſt liver.

TABLE IX. Shewing the Value of L. 1 Annuity upon a ſingle Life, according to the Probabilities of Life at Kettle, computed at 4 per Cent.

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
6–7	18.40	30	16.41	53	10.73	76	5.19
5–8	18.29	31	16.26	54	10.49	77	5.07
9	18.19	32	16.11	55	10.21	78	5.03
10	18.08	33	15.94	56	9.93	79	4.98
4–11	17.97	34	15.78	57	9.63	80	4.90
12	17.85	35	15.60	58	9.37	81	4.75
13	17.73	36	15.42	59	9.10	82	4.54
14	17.65	37	15.23	60	8.82	83	4.18
15	17.56	38	15.03	61	8.59	84	3.81
3–16	17.47	39	14.82	62	8.35	85	3.44
17	17.38	40	14.55	63	8.10	86	3.06
18	17.29	41	14.28	64	7.86	87	2.68
19	17.15	42	13.99	65	7.62	88	2.32
20	17.05	43	13.69	66	7.38	89	1.96
21	17.00	44	13.38	67	7.14	90	1.65
22	16.94	45	13.05	68	6.91	91	1.46
23	16.88	46	12.71	69	6.68	92	1.13
24	16.82	47	12.35	70	6.47	93	0.95
25	16.77	48	12.03	71	6 26	94	0.48
26	16.72	49	11.72	72	6.02	95	0.00
2–27	16 66	50	11.45	73	5.78		
28	16.61	51	11.21	74	5.55		
29	16.51	52	10.96	75	5.35		

TABLE X. Shewing the Value of L. 1 Annuity on the joint Continuance of two Lives, according to the Probabilities of Life at Kettle, computed at 4 per Cent.

Values at 4 per Cent.

10	14.38												
15	14.16	13.86											
20	13.93	13.67	13.42										
25	13.80	13.53	13.34	13.12									
30	13.62	13.42	13.25	12.98	12.73								
35	13.26	13.06	12.85	12.60	12.39	11.97							
40	12.54	12.39	12.22	12.04	11.83	11.49	11.03						
45	11.44	11.32	11.19	11.06	10.89	10.62	10.27	9.72					
50	10.22	10.11	10.01	9.91	9.79	9.60	9.33	8.92	8.38				
55	9.24	9.17	9.09	9.02	8.91	8.77	8.56	8.25	7.84	7.39			
60	8.09	8.04	7.98	7.92	7.85	7.74	7.60	7.37	7.07	6.75	6.29		
65	7.07	7.04	7.00	6.95	6.90	6.82	6.71	6.55	6.34	6.10	5.75	5.38	
70	6.08	6.05	6.02	6.00	5.95	5.89	5.82	5.70	5.55	5.38	5.15	4.88	4.53
Ages	10	15	20	25	30	35	40	45	50	55	60	65	70

Tables VII. and X. are conſtructed by the Theorem $A - \frac{n}{3m} A$, which expreſſes the number of years certain, correſponding to the value of the joint continuance of two lives, whoſe complements are n and m.

TABLE

TABLE XI. Shewing the Probabilities of Life derived from the Bills of Mortality in Edinburgh, Anno 1780.

Ages.	Living.	Dd.	Ages.	Living.	Dd.	Ages.	Living.	Dd.	Ages.	Liv.	Dd.
0	753	140	24	345	4	48	233	6	72	81	6
1	613	74	25	341	4	49	227	6	73	75	6
2	539	40	26	337	4	50	221	6	74	69	6
3	499	28	27	333	4	51	215	6	75	63	6
4	471	20	28	329	4	52	209	6	76	57	6
5	451	14	29	325	4	53	203	5	77	51	6
6	437	12	30	321	4	54	198	5	78	45	6
7	425	10	31	317	4	55	193	5	79	39	5
8	415	8	32	313	4	56	188	5	80	34	5
9	407	6	33	309	4	57	183	6	81	29	4
10	401	4	34	305	4	58	177	6	82	25	4
11	397	4	35	301	4	59	171	6	83	21	3
12	393	4	36	297	4	60	165	7	84	18	3
13	389	4	37	293	5	61	158	7	85	15	3
14	395	4	38	288	5	62	151	7	86	12	3
15	391	4	39	283	5	63	144	7	87	9	2
16	377	4	40	278	5	64	137	7	88	7	2
17	373	4	41	273	5	65	130	7	89	5	2
18	369	4	42	268	5	66	123	7	90	3	1
19	365	4	43	263	6	67	116	7	91	2	1
20	361	4	44	257	6	68	109	7	92	1	1
21	357	4	45	251	6	69	102	7	93	0	0
22	353	4	46	245	6	70	95	7			
23	349	4	47	239	6	71	88	7			

By Table XI. the number of births or burials is to that of inhabitants as 753:21968—376.5, that is as 1 : 28.67; 1 out of 22 arrives at 80 years of age, and the expectation and value of life (at 4 per cent.) are as follow.

Ages.	Expectat.	Values.	Ages	Expectat.	Values.	Ages.	Expect.	Value.
0	28.67	11.58	35	26.52	14.46	70	8.38	6.19
5	41.83	17.26	40	23.50	13 43	75	6.42	4.85
10	41.79	17 98	45	20.74	12.43	80	4.82	3.66
15	38.85	17.14	50	18.22	11.45	85	3.10	2.34
20	35.87	16.46	55	15.54	10.23	90	1.50	0.95
25	32.82	16.00	60	12.71	8.74	92	0.50	0.00
30	29.71	15.12	65	10.45	7.47			

It

It is required to find the amount of the population in the city of Edinburgh and its environs, anno 1792, excluſive of Leith. The annual number of burials = 1800; which, making allowance for adventurers, and an increaſing population, may be, to the births and ſettlers, (at 15 years of age), as 2 : 3; and theſe laſt to one another, as 3 : 1. Hence,

The annual number of births = 2000
of burials = 1800
of ſettlers = 700
their ſum = 4500

Therefore half this ſum - = 2250
multiplied by the mean expectation of life,
viz. $\frac{28.67 \times 3 + 38.85}{4}$ - - = 31.22

will produce, with the addition of 755 in the caſtle, the number of inhabitants = 71,000

Table XI. is a ſtriking evidence of the improvements in favour of life in the city of Edinburgh, during the period of half a century, from anno 1740, at which time Dr Price conſtructed a table of obſervations adapted to that city, whereby he found, agreeable to the London tables, that only 1 in 42 of all who died reached 80 years, and that the expectation of an infant's life did not exceed 18; which difference in favour of human life may ariſe from the encouragement given to trade in all its branches ſince the year 1745, whereby the bulk of the inhabitants are better fed, clothed, and lodged, have more elbow-room, and freſher air, than at any former period.

The

The Breſlaw table, (Table I.), conſtructed by Dr Halley, as alſo the Norwich and Northampton tables, publiſhed by Dr Price, agree, nearly exact in point of expectation and value, with the hypotheſis, and are ſuppoſed to be adapted to Europe in general; the London table (Table V.) correſponds to a large and populous capital; and Table VIII. and XI. contain probabilities of life, which will be found to be adapted, either to Scotland in general, or to Edinburgh in particular.

As annuities on lives are commonly bought or ſold at ſo many years purchaſe, the values aſſigned in Tables II. III. &c. may alſo be ſo reckoned; at the ſame time, they who ſell annuities have generally $1\frac{1}{2}$ or 2 years purchaſe of more value than ſpecified in the tables, from purchaſers whoſe age is 20 years and upwards.

Ex. A perſon reſiding in London, aged 30 years, wiſhes to purchaſe an annuity for life of L. 100 per annum; what muſt he pay for it, reckoning intereſt at 4 per cent.?

Here $13.31 \times 100 = $ L. 1331, the purchaſe-money.

An annuity begins to run from the date of the event which intitles it: and in the caſe of a ſimple annuity paid in money, the annuity ceaſes at the term prior to the annuitant's death; but in the caſe of an annuity ſecured by land, the heirs of the annuitant are intitled to payment to the laſt moment of his life. In this caſe, by increaſing the numbers in Table II. by .24 of a years purchaſe, to 54 years of age, by .33, from 54 to 70, and by .44 from 70 and upwards, we ſhall obtain the preſent values nearly of an annuity (ſecured by land) of L. 1 per annum on a

ſingle

ſingle life, ſuppoſing the decrements of life to be equal.

As the real law, according to which human life waſtes, approaches much nearer to an arithmetical than to a geometrical progreſſion, all thoſe theorems of Mr de Moivre are rejected which are built upon the principle, "That the "decrements of life are in a conſtant *Ratio*," ſeeing they give the value of life too ſmall.

PROBLEM IV.

Suppoſing the decrements of life to be equal, it is required, to find the value of L. 1 annuity upon the longeſt of two lives, whoſe reſpective complements are n and m, (m being the greater number), at a given rate of intereſt.

64. $\frac{n-1}{n}$ is the probability that a life, whoſe complement is n, will exiſt one year, and $\frac{1}{n}$ the chance of its failing in that time; but the ſum of theſe two fractions is equal to unity: Therefore having one of them given, we may, by ſubtraction, find the other. Now the probabilities of the continuance of thoſe lives for one year, are reſpectively $\frac{n-1}{n}$ and $\frac{m-1}{m}$; and the probabilities of their ſeverally failing are $1-\frac{n-1}{n}$ and $1-\frac{m-1}{m}$; conſequently the probability of their both failing in that time, will be $\overline{1-\frac{n-1}{n}} \times \overline{1-\frac{m-1}{m}}$, which, ſubtracted from unity, leaves the probability of there

there being one of them, at leaſt, exiſting at the year's end. But,

$$\overline{1-\frac{n-1}{n}} \times \overline{1-\frac{m-1}{m}} = 1-\frac{n-1}{n}-\frac{m-1}{m}+\frac{n-1\times m-1}{nm},$$

which, taken from unity, leaves $\frac{n-1}{n}+\frac{m-1}{m}-\overline{\frac{n-1\times m-1}{nm}}$, the firſt payment of the annuity in the parts of L. 1.

In like manner, $\frac{n-2}{n}+\frac{m-2}{m}-\overline{\frac{n-2\times m-2}{nm}}$ is the ſecond payment of the annuity, &c.; hence, the required annuity may be expreſſed in the following manner:

$$\frac{n-1}{nR}+\frac{m-1}{mR}-\overline{\frac{n-1\times m-1}{nmR}},$$

$$\frac{n-2}{nR^2}+\frac{m-2}{mR^2}-\overline{\frac{n-2\times m-2}{nmR^2}},$$

$$\frac{n-3}{nR^3}+\frac{m-3}{mR^3}-\overline{\frac{n-3\times m-3}{nmR^3}}, \text{ &c.}$$

But the ſeries in the firſt and ſecond columns, are the values of annuities on the ſingle lives, whoſe complements are n and m; and the third column is the value of an annuity upon the joint lives of the two perſons; therefore,

If, from the ſum of the values of the ſingle lives of two perſons, the value of their joint lives be taken, the remainder will be the value of an annuity on the longeſt of them.

Or thus: Let x and y repreſent the reſpective probabilities of the two lives continuing for one year; then,

$\overline{1-x}\times\overline{1-y}=1-x-y+xy$, the probability of the two lives failing in that time, ſubtracted from

unity, leaves $x+y-xy$, the probability of one, at leaſt, of the two lives ſurviving the year; and by ſubſtituting the values of the lives in place of the probabilities thereof, (N and M being the values of the ſingle lives, and $\overline{NM}$, that of the two joint ones,) then $N+M-\overline{NM}$ will be the value of an annuity upon the longeſt of them.

Ex. It is required, to find the value of an annuity of L. 1 on the longeſt life of two perſons, whoſe reſpective ages are 35 and 25 years, allowing intereſt at 4 per cent. Here,

$$N+M=29.297$$
$$-\overline{NM}=11.095$$

their difference = L. $\overline{18.202}$ is the value required.

If the two lives are equal, (N "being the value of the two joint ones,) then $2N-N$" will be the value of an annuity on the longeſt of them.

PROBLEM V.

Suppoſing the decrements of life to be equal, it is required to find the value of L. 1 annuity upon the longeſt of three lives, (the values of the ſingle ones being given), at a given rate of compound intereſt.

65. By following a proceſs ſimilar to that obſerved in the laſt problem, it might be ſhewn, that the value of an annuity upon the longeſt of three lives is equal to

+ the values of the three ſingle ones,

— the three values of the joint lives, taken two and two,

+ the value of the three joint lives. Or thus:

Let

Let x, y, and z repreſent the reſpective probabilities of the continuance of the three lives for one year; then,

$\overline{1-x} \times \overline{1-y} \times \overline{1-z} = 1-x-y-z+xy+xz+yz-xyz$, will repreſent the probability of the three lives failing in one year, which ſubtracted from unity, leaves,

$x+y+z-xy-xz-yz+xyz$, the probability of one, at leaſt, of the three lives outliving the year; and by ſubſtituting the values of the lives in place of the probabilities thereof, ($\overline{NMF}$ being the value of the three joint lives,) then $N+M+F-\overline{NM}-\overline{NF}-\overline{MF}+\overline{NMF}$ = value of the annuity.

Ex. Let the three ſingle lives be worth 13, 14, and 15 years purchaſe; ſought, the value of an annuity upon the longeſt of them. Here,

$$13+14+15+8.7=50.7$$
$$-10.1-10.5-10.9=31.5$$

the value required $=19.2$

PROBLEM VI.

To determine the value of an annuity of L. 1, granted for a given term of years, denoted by n, on the contingency of its ceaſing upon the extinction of an aſſigned life, whoſe complement is m, at a given rate of compound intereſt.

66. Since every payment of the annuity depends upon the continuance of the aſſigned life, whoſe complement is m, it will be worth n terms of the ſeries, viz.

$\frac{m-1}{mR}+\frac{m-2}{mR^2}+\frac{m-3}{mR^3}(n)$ which may be divided into two other ſeries, viz.

$\frac{m}{m} \times \overline{\frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} (n)} = P.$ (47)

$-\frac{1}{m} \times \overline{\frac{1}{R} + \frac{2}{R^2} + \frac{3}{R^3} (n)} = \frac{1}{m} \times \overline{\frac{RP}{r} - \frac{np}{r}}$ Prob. 1ſt.

Hence $P - \overline{\frac{RP}{mr} - \frac{np}{mr}} = P - \frac{RP-np}{mr}$ is the value required.

Ex. *A*, aged 40 years, engages to pay to *B* or his heirs, an annuity of L. 1 for 15 years, if he (*A*) ſhall live ſo long. Sought, the value of *B*'s annuity, reckoning intereſt at four per cent. Here, $n=15$, $m=46$, $P=11.118$, and $p=.55526$. Hence, $11.118 - \frac{3.234}{1.84} = 9.36$, is the value of *B*'s annuity.

TABLE XII. Shewing the Value of L. 1 Annuity for a certain Number of Years, depending upon the contingency of a given Life.

Ages, computing intereſt at 4 per cent.

Yrs.	10	15	20	25	30	35	40	45	50	55	60	65	70
5	4.28	4.27	4.25	4.24	4.22	4.20	4.17	4.14	4.09	4.03	3.95	3.83	3.64
10	7.56	7.52	7.47	7.42	7.36	7.29	7.20	7.09	6.94	6.76	6.50	6.11	5.49
15	10.05	9.97	9.89	9.79	9.67	9.53	9.36	9.14	8.87	8.50	8.00	7.27	
20	11.95	11.83	11.70	11.54	11.36	11.14	10.87	10.54	10.12	9.56	8.78		
25	13.36	13.20	13.02	12.81	12.56	12.26	11.89	11.43	10.85	10.08			
30	14.42	14.22	13.98	13.71	13.39	13.01	12.55	11.97	11.23				
35	15.20	14.95	14.66	14.34	13.96	13.50	12.93	12.24					
40	15.76	15.48	15.15	14.77	14.32	13.79	13.13						
45	16.27	15.8[illegible]	15.56	15.05	14.54	13.92							
50	16.45	16.0[illegible]	15.68	15.21	14.65								

Mr Simpſon, in his ſelect exerciſes, p. 262, is very defective in the conſtruction of this table; which yet may eaſily be calculated from the relation which the above theorem hath to $P - \frac{RP-np}{nr} = N$, the value of the ſingle life, whoſe complement is n, Prob. I. Thus, for

Ex.

Ex. A, aged 25, enjoys an annuity for the ſpace of 20 years, if he ſhall live ſo long; it is required, to find the preſent value of ſaid annuity, reckoning intereſt at 4 per cent. Here $n=20$, and $m=61$, the complement of the life; therefore,

From the value of an annuity for 20 years = 13.590
Subtract the value of a life aged 66 years = 7.333

their difference = 6.257
multiplied by n = 20

their product divided by m = 61) 125.140 (13.59 / 2.05

There remains the anſwer ſought = 11.54

A Queſtion of Inſurance on Lives.

A, aged 16 years, is aſſured of the ſum of L. 500, in caſe he ſhall live to be 21 years of age complete; it is required, to find what premium he ought to pay to inſure his life during theſe 5 $=n$ years, upon that ſum.

Let $\frac{a}{b}=\frac{65}{70}$, $\frac{n}{b}=\frac{5}{70}$ = A's chance of { living 5 years. / dying in that time.

But A's intereſt in L. $500=\frac{a}{b}\times 500=$ L. 464.3, their difference, viz. L. 35.7, is the premium of inſurance; or A may lay his account to loſe $\frac{n}{b}\times 500=$ L. 35.7, the ſame premium.

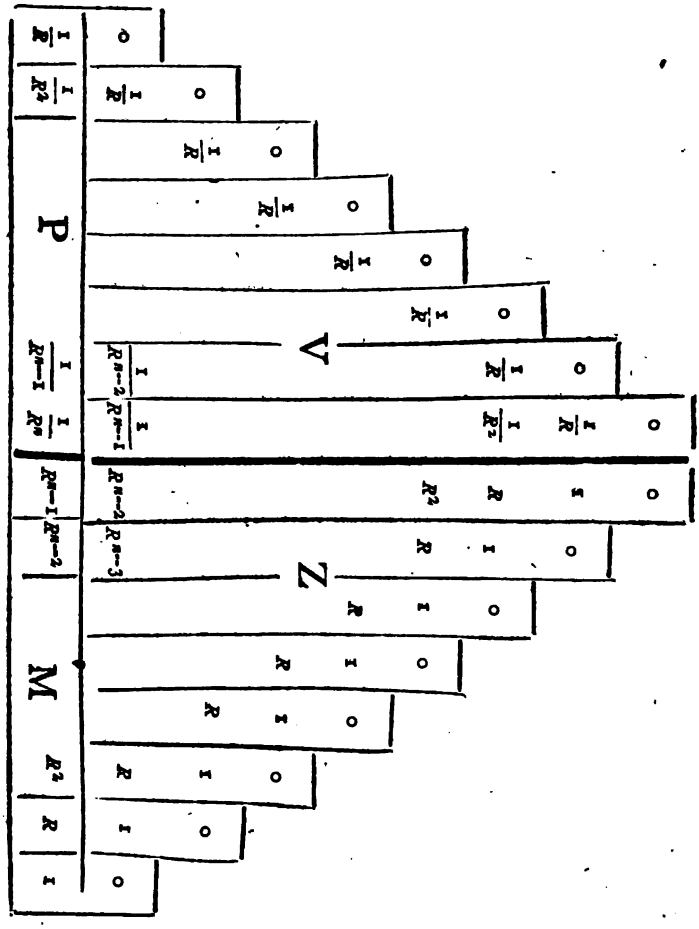
If this premium be paid per advance, at a given rate; then $\frac{35.7}{R^n}=$ L. 29.34, its preſent value at 4 per cent.

If A ſells his intereſt in this L. 500, at a given

ven rate; then $\frac{4643}{R^n} = L.\ 382.45$, the value at 4 per cent. *See* Prob. XV. Ex. 1ſt and 2d.

Supplement to Problem I. II. III. IV. V. VI. *Chap.* VIII.

ART. 1. To find, by a geometrical figure, the value or amount of L. 1, or of L. 1 annuity, upon a ſingle life, whoſe complement is n, at a given rate of intereſt.



1ſt,

1*ſt*, Seeing, that out of n perſons alive, at a given age, whoſe complement is n, one is ſuppoſed to fail yearly, till they are all exhauſted at the age of 86; the two triangles V and Z, which contain the value or amount of L. 1 annuity for $1+2+3\ldots+n-1$ years, will repreſent the value or amount of L. 1 annuity upon n lives, whoſe common complement is n, at a given rate; and the two bottom columns P and M, which contain the value or amount of L. 1 for $1+2+3\ldots+n$ years, will ſhow the value or amount of L. 1 annuity for n years, at the ſame rate; hence,

let V or Z = val. or am. of L. 1 annuity for $1+2+3\ldots+n-1$ years,
P or M = — or — of ——— for n years; then,

$$\left.\begin{matrix} V \text{ or } Z \\ P \text{ or } M \end{matrix}\right\} \div n = \begin{cases} \text{val. or am. of L. 1 annuity} & \begin{cases} \text{upon a ſingle life.} \\ \text{whoſe complet. is } n. \end{cases} \\ \text{— or — of L. 1} \end{cases}$$

2*dly*, But ſeeing that $\frac{1-rN}{R}$ is alſo = value, and $1+rs$ = amount of L. 1 upon a ſingle life, whoſe complement is n, at a given rate:

$$\text{Hence } N = \frac{n-RP}{rn} = \text{value} \begin{cases} \text{of L. 1 annuity upon a life, whoſe complement is } n. \end{cases}$$
$$S = \frac{M-n}{rn} = \text{amount}$$

Ex. It is required, to find the value or amount of L. 1, or of L. 1 annuity, upon a life aged 36 years, at 4 per cent. Here $n=50$, $P=21.482$, $M=152.667$, $R=1.04$, and $r=.04$. Hence,

$$\left.\begin{matrix} n-RP \\ M-n \end{matrix}\right\} \div rn = \begin{cases} 13.829 = N, \text{ the va.} & \begin{cases} \text{of L. 1 annuity.} \end{cases} \\ 51.34 = s, \text{ the am.} \end{cases}$$
$$\left.\begin{matrix} P \\ M \end{matrix}\right\} \div n = \begin{cases} .4296 = \text{va.} & \begin{cases} \text{of L. 1, upon a life, age 36 years.} \end{cases} \\ 3.053 = \text{am.} \end{cases}$$

ART.

ART. 2. To find the value or amount of L. 1 or L. 1 annuity, upon the joint continuance of 2, 3, 4. . . . m lives, at a given rate of intereſt. Here,

1ſt, Let n repreſent the mean complement of all the lives,

x ——— the expectation of their joint continuance,

X ——— the expectation of the longeſt liver,

P or M —— the value or amount of L. 1 annuity for x years,

$\mathfrak{Q}$ or A —— the value or amount of L. 1 annuity upon a life whoſe complement is x, and

p or a —— the value or amount of L. 1 for x years; then,

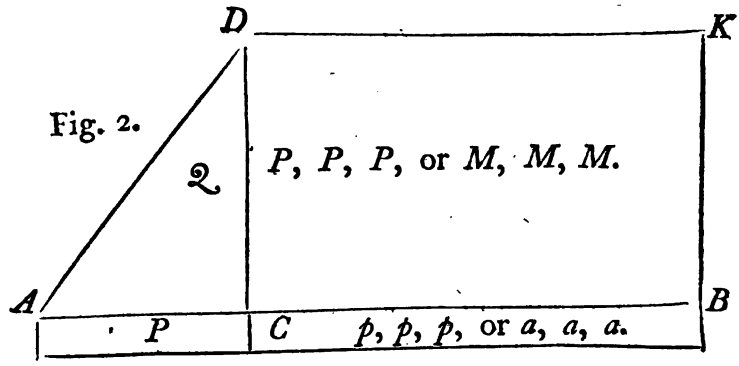


Fig. 2.

In Fig. 2. put $AB=n$, AC or $CD=x$, of conſequence CB or $DK=n-x=X$, and complete the rectangle BD, and triangle $\mathfrak{Q}$. Now, ſeeing that out of n lives, whoſe mean complement is n, one will fail yearly during x years, $DB+\mathfrak{Q}=\overline{n-x}\times P+$value of a life whoſe complement is x multiplied

multiplied by x, will repreſent the value of L. 1 annuity upon n lives for x years; and the bottom column, viz. $\overline{n-x} \times p + P$, will ſhew the value of L. 1 on n lives for x years; hence,

$$\left.\begin{matrix} n-x \\ =X \end{matrix}\right\} \left.\begin{matrix} \overline{\times P \text{ or } M + x \times \mathfrak{Q} \text{ or } A} \\ \times p \text{ or } a + P \text{ or } M \end{matrix}\right\} \div n = \left\{\begin{matrix} \text{Val. or am. of} \\ \text{L. 1 annuity.} \\ \text{Val. or am. of} \end{matrix}\right.$$

L. 1 during the joint continuance of the lives.

Ex. It is required, to find the value or amount of L. 1, or of L. 1 annuity, upon the joint continuance of three lives, whoſe reſpective ages are 40, 36, and 26 years, reckoning intereſt at 4 per cent. Here,

$n=52$, $x=12.88$, $X=39.12$, $P=9.913$, $M=16.434$, $\mathfrak{Q}=5$, $A=6.9$, $p=.6035$, and $a=1.657$. Hence,

$$\left.\begin{matrix} \overline{387.796+64.4} \\ 642.898+88.87 \\ 23.609+\ 9.913 \\ 64.822+16.434 \end{matrix}\right\} \div 52 = \left\{\begin{matrix} 8.696 = \text{val.} \\ 14.072 = \text{am.} \\ .6446 = \text{val.} \\ 1.563 = \text{am.} \end{matrix}\right. \left.\begin{matrix} \text{of L. 1} \\ \text{ann.} \\ \text{of L. 1} \\ \text{du-} \end{matrix}\right.$$

ring the joint continuance of theſe three lives.

2d, In the caſe of m equal lives, whoſe common complement is n, $x=\frac{n}{m+1}$, and the co-efficients of P and $\mathfrak{Q}$, viz. $\frac{n-x}{n}+\frac{x}{n}=\frac{mn}{mn+n}+\frac{n}{mn+n}$. Hence,

$$\left.\begin{matrix} \overline{m \times P \text{ or } M + \mathfrak{Q} \text{ or } A} \\ -\times p \text{ or } a + \frac{P \text{ or } M}{n} \end{matrix}\right\} \div m+1 = \left\{\begin{matrix} \text{val. or am. of L. 1 ann.} \\ \text{value or amount} \end{matrix}\right.$$

of L. 1 upon the joint continuance of theſe lives.

In the caſe of 4, 5, . . . m unequal lives, find their mean age, and conſider it as the age of ſo many equal lives.

ART. 3d, To find the preſent value or amount of L. 1, or of L. 1 annuity, during the longeſt liver of two or more lives, at a given rate of intereſt.

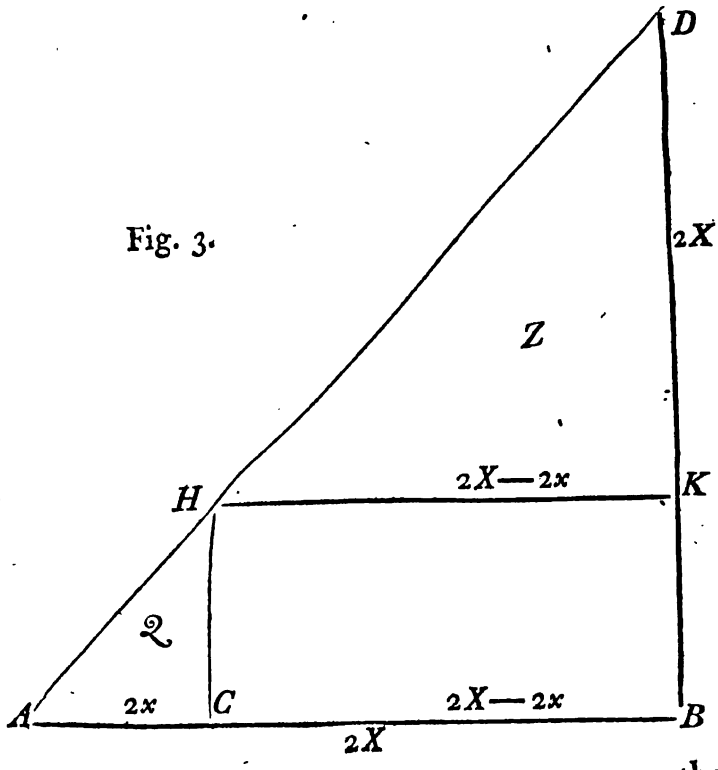
1ſt, Let n repreſent the mean complement of all the lives,

x ——— the expectation of their joint continuance, and

$-X$ ——— the expectation of the longeſt liver; then

In Fig. 3. make AB or $BD = 2X$, AC or $CH = 2x$, of conſequence, CB or $HK = 2X - 2x$ = twice

Fig. 3.



the

the expectation of ſurvivorſhip, and complete the triangle *DAB*. Here,

The triangle *DAB* — $2X$ = val. or am. of longeſt liver.
—— $\mathfrak{Q}$ — $2x$ = val. or am. of joint continuance.
—— Z — $2X - 2x$ = value or amount of ſurvivorſhip, computed at the diſſolution of the joint lives, which value, $\times \frac{1-r\mathfrak{Q}}{R}$, and added to that of $\mathfrak{Q}$, will alſo give the value of the longeſt liver, nearly exact. And the amount of $\mathfrak{Q}$, $\times 1+rs$, and added to that of Z, will likewiſe ſhew the amount of the longeſt liver, nearly.

Ex. It is required, to find the value and amount of L. 1 annuity upon the longeſt liver of three lives, whoſe reſpective ages are 40, 36, and 26 years, at 4 per cent. Here,

$n=52$, $x=12.88$, $X=39.12$; hence,

The triangle *DAB* by Art. 1. $=17.076$ = val. $=138.9$ = am. of L. 1 annuity upon the longeſt liver of theſe lives.

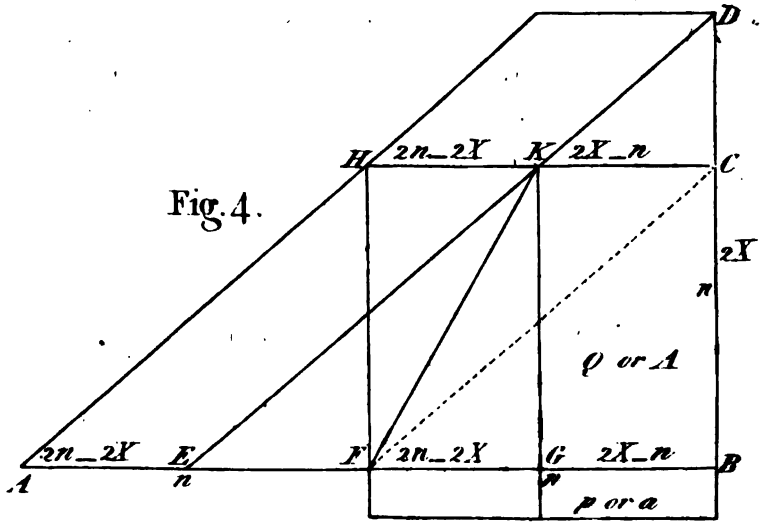
2d, To the value of $\mathfrak{Q} =$ - 8.962
add that of Z, $\times \frac{1-r\mathfrak{Q}}{R} =$ - 8.813
Their ſum = value of the longeſt liver = 17.775

And to the amount of $Z = s =$ - 56.42
add that of $\mathfrak{Q}$, $\times 1+rs =$ - 56.78
Their ſum = amount of the longeſt liver = 113.20

The value or amount of L. 1 annuity upon the longeſt liver, found thus, would be exact, were it not for this circumſtance, that the triangle *DAB* ſtretcheth with its vertex into the regions of improbability, where human life cannot reach, and of conſequence gives a value rather ſmall, and an amount too great; to remedy this, obſerve the following Method.

3d, Let n, x, and X repreſent as above,
Put P or M = value or amount of an annuity for n years,
Q or A = value or amount of a life whoſe complement is n, and
p or a = value or amount of L. 1 for n years; then

Fig. 4.



In Fig. 4. put $AB=2n$, EB or $BD=2X$, FB or $BC=n$, and complete the ſquare HB, the rectangle KB, and the triangles DEB, and CFB, and join KF. Now it is evident, that the value or amount of L. 1 annuity upon the longeſt liver of any number of lives, where the mean complement of all the lives is n, will vibrate betwixt the triangle $\mathcal{Q}$ and ſquare HB; and that the line KF, and point K, will limit theſe values; hence,

$$\left.\begin{array}{l}\overline{\begin{array}{l}2X-n\\=X-x\end{array}}\times P \text{ or } M+\overline{\begin{array}{l}2n-2X\\=2x\end{array}}\times \mathcal{Q} \text{ or } A\\ \text{——}\times p \text{ or } a+\frac{2x}{n}\times P \text{ or } M\end{array}\right\}\div n=\left\{\begin{array}{l}\text{val. or am. of L. 1 annuity.}\\ \text{val. or am. of L. 1 upon the longeſt liver of theſe lives.}\end{array}\right.$$

Ex. In the example to this article, $n=52$, $x=12.88$, $X=39.12$, $P=21.748$, $M=167.165$, $\mathcal{Q}=14.126$, $A=55.37$, $p=.1301$ and $a=7.687$. Hence,

$$\left.\begin{array}{l}570.667+363.686\\386.41+1426.33\\3.414+10.774\\201.707+82.811\end{array}\right\}\div 52=\left\{\begin{array}{ll}17.968=\text{val} & \text{of L. 1 annuity.}\\ 111.783=\text{am.} & \\ .2728=\text{val.} & \text{of L. 1, during the longeſt}\\ 5.471=\text{am.} & \end{array}\right.$$

liver of three lives, whoſe reſpective ages are 40, 36, and 26 years, at 4 per cent.

4th, In the caſe of m equal lives, whoſe common complement is n; $x=\frac{n}{m+1}$, $X=\frac{mn}{m+1}$,

$P=$value of L. 1 annuity for n years, &c, and the co-efficients of P and $\mathcal{Q}$, viz. $\frac{2X}{n}\,\frac{n}{}+\frac{2n-2X}{n}$ $=\frac{mn-n}{mn+n}+\frac{2n}{mn+n}$. Hence,

$$\left.\begin{array}{l}\overline{m-1}\times\overline{P \text{ or } M+2\times \mathcal{Q} \text{ or } A}\\ \text{——}\times p \text{ or } a+\frac{2}{n}\times P \text{ or } M\end{array}\right\}\div m+1=\left\{\begin{array}{l}\text{value or amount of L. 1 annuity.}\\ \text{value or amount of L. 1 upon the longeſt liver of theſe equal lives.}\end{array}\right.$$

Ex.

Ex. Sought, the value of the longeſt liver of 4 equal lives, whoſe common age is 30 years, at 4 per cent. Here,

$m=4$, $P=22.2198$, and $\mathfrak{Q}=14.684$. Hence,

$\frac{3\times 22.2198+2\times 14.684}{5}=19.205=$ value required.

In the caſe of 4, 5 ... m unequal lives, find their mean age, and conſider it as the age of ſo many equal lives.

5th, If the value, &c. of an annuity be required, which ſhall continue during the longeſt liver of q out of m lives of a given age and at a given rate; here $X=\frac{qn}{m+1}=$ expect. of the longeſt liver.

$P=$ value of an annuity for n years, &c. and

$$\left.\begin{array}{l}\overline{q-\frac{m+1}{2}}\times P \text{ or } M+\overline{m+1-q}\times \mathfrak{Q} \text{ or } A\\ \overline{\quad\quad}\times p \text{ or } a+\frac{m+1-q}{n}\times P \text{ or } M\end{array}\right\}\div\frac{m+1}{2}=\left\{\begin{array}{l}\text{value or am. of L. 1 ann.}\\ \text{value or amount of L. 1}\end{array}\right.$$

during the longeſt liver of q out of m lives.

Ex. It is required to find the value of L. 1 annuity upon the longeſt liver of 10, out of 15 equal lives, whoſe common age is 22 years, at 4 per cent. Here,

$m=15$, $q=10$, $P=22.968$ and $\mathfrak{Q}=15.669$; hence, $\frac{2\times 22.968+6\times 15.669}{8}=17.494$, the value required.

When q is leſs than $\frac{m+1}{2}$, the firſt term is negative.

ART. 4. To find the value or amount of L. 1, or of L. 1 annuity, for a term of years denoted by n, but ſubject to failure on the contingency of

of an aſſigned life, whoſe complement is m, at a given rate of intereſt.

In fig. 2. ſuppoſe $AB=m$, AC or $CD=n$, and let the rectangle DB and triangle $\mathfrak{Q}$ be completed; now, ſeeing that out of m lives, whoſe complement is m, one is ſuppoſed to fail yearly during n years, let

P or M = value or amount of L. 1 annuity for n years.

$\mathfrak{Q}$ or A = —— or —— of an annuity upon a life whoſe complement is n.

p or a = —— or —— of L. 1 for n years; then

$$\left.\begin{array}{l}\overline{m-n\times P \text{ or } M+n\times \mathfrak{Q} \text{ or } A}\\ \text{---}\times p \text{ or } a+P \text{ or } M\end{array}\right\}\div m=\left\{\begin{array}{l}\text{val. or am. of L. 1 annuity.}\\ \text{--- or --- of L. 1 for a}\end{array}\right.$$

term of years denoted by n, on the contingency of an aſſigned life whoſe complement is m.

Ex. It is required, to find the value or amount of L. 1, or of L. 1 annuity, during the term of 15 years, yet liable to be demanded, or ſubject to failure, on the contingency of a life aged 36 years, at 4 per cent.

Here $n=15$, $m=50$, $P=11.1184$, $M=20.0236$, $\mathfrak{Q}=5.728$, $A=8.38$, $p=.5553$, and $a=1.8$. Hence,

$$\left.\begin{array}{l}389.144+85.92\\ 700.826+125.70\\ 19.435+11.118\\ 63.000+20.024\end{array}\right\}\div 50=\left\{\begin{array}{ll}9.501=\text{value} & \\ 16.53=\text{amount} & \text{of L. 1 annuity.}\\ .611=\text{value} & \text{of L. 1 during the}\\ 1.66=\text{amount} & \text{term of 15 years,}\end{array}\right.$$

on the contingency of a life aged 36.

REMARKS.

REMARKS.

1*ſt*, The values, &c. acquired by Art. 1. and 4. are perfectly accurate, and agree exactly with the algebraic inveſtigations, Prob. I. and VI.

2*d*, The value, &c. of the longeſt liver, found by Art. 3. is diſcovered with much leſs labour, and is nearly as accurate as the reſults of Prob. IV. and V.

3*d*, The value, &c. of 3, 4 . . . *m* joint lives, Art. 2. is as perfect as the nature of the ſcience will admit. The value of 2 joint lives is nearly correct at the early periods of life; but it declines as life advances, in ſuch a manner, that, at the mean age of 40, it would require 1 year's purchaſe to be added to $\mathfrak{Q}$ at 4 per cent. to make it correſpond with Prob. II. and III.

4*th*, The theorems contained in theſe articles, (excepting caſe 2d, Art. 1.), may with equal propriety be applied to life-annuities at ſimple intereſt, by making uſe of tables of ſimple, in place of thoſe of compound intereſt. They may alſo be uſed in finding the value of life from a table of obſervations.

Ex. 1ſt, To find, from the Fife Table of Obſervations, the value of the joint continuance of two lives, whoſe reſpective ages are 30 and 25 years, at 4 per cent. Here,

N=

$N = 32.54$	$x =$ N° failing in $\overline{NM}$ years $= 15.84$
$M = 34.99$	$n - x \quad = 51.69$
Sum $= n = 67.53$	$P =$ val. of ann. for $\overline{NM}$ yrs $= 14.63$
$\overline{NM} = 22.45$	$Q =$ val. of a life ——— $= 8.05$ and

$\frac{51.69 \times 14.63 + 15.84 \times 8.05}{67.53} = 13.08$, value ſought; which agrees nearly exact with the reſult of the theorm $\frac{n}{i} A$.

Ex. 2d, To find the value of the longeſt liver of two lives, whoſe reſpective ages are 40 and 30 years, at 4 per cent. from the ſame table. Here,

$N = 25.83$	$2X - n \quad = 20.37$
$M = 32.54$	$2n - 2X \quad = 38.00$
Sum $= n = 58.37$	$P =$ val. of ann. for n years $= 22.466$
$\overline{NM} = 19.00$	$Q =$ val. of a life aged 35 $= 15.60$
Diff. $= X = 39.37$	Hence,

$\frac{20.37 \times 22.466 + 38 \times 15.6}{58.37} = 18.00$, value; which is the ſame with the reſult of the theorem $X - \frac{x}{N} \times d$, Prob. III. Caſe 2.

Of REVERSIONS.

PROBLEM VII.

67. TO find the value of the reverſion of an eſtate in fee ſimple, after one or more lives of given ages, at a given rate of compound intereſt. Here,

$$\text{Or, } \frac{1}{r} - \overline{N} \;\Big|\; \frac{1}{r} - \overline{\frac{1}{r} - \frac{RP}{nr}} = \frac{RP}{nr}$$

$\Big|$ = the reverſion, in perpetuity, of L. 1 per annum, after a life whoſe value is N, at a given rate of intereſt.

Ex. Sought, the value of a reverſion, in perpetuity, of L. 1 per annum, after the life of a perſon aged 43 years, allowing intereſt at 4 per cent. Here,

$$\frac{1}{r} - N = \frac{RP}{nr} = 25 - 12.683 = 12.317, \text{ the reverſion.}$$

In like manner,

$$\left.\begin{array}{l} \frac{1}{r} - \overline{NM} \\ \frac{1}{r} - \overline{\overline{NMF}} \end{array}\right\} = \text{a reverſion in perpetuity after} \left\{\begin{array}{l} \text{2 joint lives.} \\ \text{3 joint lives.} \end{array}\right.$$

If the purchaſe is diſcharged in yearly payments during the continuance of the life or lives in queſtion; in this caſe, the reverſion, divided by the value of the life or lives, will quot the annual payments.

Ex.

Ex. A and B, whoſe reſpective ages are 35 and 25 years, ſell the reverſion of an eſtate whoſe rent is L. 150 per annum, for yearly payments during the longeſt of their lives; ſought, theſe payments, reckoning intereſt at 4 per cent. Here,

$$25 - 18.202 = 6.798$$
multiplied by 150

And the product divided by 18.2) 1019.7 (L. 56, the annual payment.

If the firſt of the annual payments be made per advance, the above diviſor, viz. 18.2, increaſed by unity, will quote L. 52.1, the annual payments.

Cor. 1ſt, To find the preſent value of L. 1, due at the deceaſe of a perſon of a given age, at a given rate of intereſt.

L. 1, or any other ſum of money, may be conceived to be the preſent worth of a perpetuity, whoſe rent is equal to the intereſt of that ſum for one year; hence, the reverſion of a fee-ſimple equal to that intereſt, after the given life, will be the value required. Thus,

$\frac{1}{r} - N \times r = 1 - rN = \frac{RP}{nr} \times r = \frac{RP}{n}$ would be the value required, were it not for one circumſtance, viz. that, from the nature of the calculation, the time in which the firſt yearly payment of a reverſionary annuity becomes due, is the end of the year in which the event happens that entitles it. This is alſo the time when a reverſionary ſum becomes due. But an annuity, the firſt payment of which is to be made at the ſame time with

another payment of a ſum in hand ſufficient to buy an equal annuity, is worth one year's purchaſe more than the ſum. Hence,

$\frac{1-rN}{R} = \frac{p}{n}$ = the value of L. 1 due at the deceaſe of a perſon whoſe complement of life is n. Or thus:

$\frac{1}{n}$ = the preſent probability, that a life whoſe complement is n will fail in any one aſſignable year of its duration; (ſee Prob. XIX. caſe 1.) Hence,

$\frac{1}{nR} + \frac{1}{nR^2} + \frac{1}{nR^3}\,(n) = \frac{p}{n}$ = the preſent value of L. 1, payable at the failure of a life whoſe complement is n; which, multiplied by s, a ſum due at the cloſe of a given life, will produce the preſent value thereof.

Ex. Let there be a legacy of L. 100 $= s$, due at the deceaſe of a perſon aged 60 years; ſought, its preſent value, at 4 per cent. Here,

$s \times \frac{p}{n} = 100 \times .6147 =$ L. 61.47, the value required.

Cor. 2d, To find what ſum ought to be paid, on the deceaſe of a perſon of a given age, in conſideration of L. 1 now received. It is evident, that the reciprocal of the former corollary will be the anſwer in this. Thus,

$\frac{R}{1-rN} = \frac{n}{p}$ = the ſum which ought to be paid at the deceaſe of a perſon whoſe complement of life is n, for L. 1 now received, which, multiplied by

p,

p, any preſent ſum, will produce s, the amount thereof at the cloſe of a given life. Or thus:

Seeing $\frac{1}{n}+\frac{R}{n}+\frac{R^2}{n}$, $(n)=\frac{M}{n}$, the amount of L. 1 annuity for n years certain divided by n, and expreſſes the amount of L. 1, payable at the failure of a life whoſe complement is n; therefore,

$$p\times\frac{M}{n}=s.$$

Ex. Let p=L. 100, the age 60 years, and the rate of intereſt 4 per cent.: Here, $n=26$, $M=44.3117$, and $p\times\frac{M}{n}=100\times 1.7043=$ L. 170.43$=s$, which is the ſame with the reſult from the theorem $p\times\overline{1+rs}$ of Table IV.

PROBLEM VIII.

68. Suppoſing the decrements of life to be equal, it is required, to find the value of the reverſion of an annuity, which is to continue during the life of a perſon whoſe complement is m, after the deceaſe of the preſent poſſeſſor, whoſe complement of life is n, at a given rate of intereſt.

$1-\frac{n-1}{n}$, $1-\frac{n-2}{n}$, &c. are the probabilities of the poſſeſſor's dying in the firſt, ſecond, &c. years; and $\frac{m-1}{m}$, $\frac{m-2}{m}$, &c are the probabilities of the expectant's living to the end of the firſt, ſecond, &c. years. Hence $\frac{m-1}{m}\times\overline{1-\frac{n-1}{n}}$, $\frac{m-2}{m}\times\overline{1-\frac{n-1}{n}}$, &c. are the probabilities that the expectant

tant will receive the firſt, ſecond, &c. yearly payment, which probabilities being expanded by multiplication, and their preſent values found, will give the value of the reverſion, viz.

$$\frac{m-1}{mR} - \frac{\overline{m-1} \times \overline{n-1}}{mnR},$$

$$\frac{m-2}{mR^2} - \frac{\overline{m-2} \times \overline{n-2}}{mnR^2}, \text{ &c.}$$

But the ſeries in the firſt column is the value of the expectant's life, whoſe complement is m; and the ſecond column contains the value of the joint lives of the poſſeſſor and expectant. Therefore, if, from the value of the expectant's life, the value of the joint lives be taken, the remainder will be the value of the reverſion.

Or thus: Let $\overline{1-x} \times y = y - xy$ be the probability that the expectant will outlive the poſſeſſor the firſt year; then, by ſubſtituting the values of the lives in place of the probabilities thereof, $M - \overline{NM}$ will be the preſent value of the expectant's life after the death of the poſſeſſor, or the value of the reverſion required.

Ex. Let the reſpective ages of the expectant and poſſeſſor be 25 and 30 years, and the rate of intereſt 4 per cent. then $M = 15.318$, $N = 14.684$ and $\overline{NM} = 11.468$; hence, $M - \overline{NM} = 3.85$, the value of the reverſion.

Of

Of a Widows Scheme.

69. THE reſolution of the ſeveral queſtions reſpecting a reverſionary annuity to one life after two, is derived from the above theorem, viz. $M - \overline{NM}$ = the value of the reverſion. There are two caſes of ſuch an annuity, which differ from or agree with one another in a few particulars.

Caſe 1. A and B, whoſe reſpective ages are 30 and 25 years, agree to pay a certain ſum in hand, or a certain annual rate during the continuance of their joint lives, in order to purchaſe an annuity to B after A's deceaſe. Sought theſe, reckoning intereſt at 4 per cent.

$M - \overline{NM} = 3.85$ = ſum in hand, or preſent value.

$\frac{M-\overline{NM}}{\overline{NM}} = \text{L.}0.3357$ = annual rate payable during their joint lives to ſecure L. 1 annuity to B.

In this caſe, the payment of the rates ceaſes at the cloſe of the joint lives, whether A be alive or not, and B is ſuppoſed to be the ſurvivor.

Had this caſe been conditional, viz. "if B ſhall ſurvive A," the ſum in hand and annual rate muſt be diminiſhed by $\frac{41.57}{61.00} = .681$, the probability of B's ſurviving the diſſolution of marriage.

Had part of the preſent value of the annuity, ſuch as L. 1.5 been paid in ready money, the remainder, divided by $\overline{NM}$, quots the annual rate; and were the rates payable per advance, $\overline{NM}+1$ will be the diviſor.

Caſe 2d, Is that of a widows ſcheme, where either

either a certain ſum in hand, or an annual rate during the life of A, is paid, in order to ſecure a proviſion for B, after A's deceaſe, or, failing B, ſo many years purchaſe of her annuity to their children.

This caſe differs *in toto* from the conditional one mentioned above; and in part from Caſe 1ſt, in this, that the annual rate is paid during the life of A; but agrees with it in ſuppoſing that either a widow or children are left a burden upon the ſcheme. Therefore,

Let the reſpective ages of A and B be 30 and 25 years, and the rate of intereſt 4 per cent; then, $M - \overline{NM} =$ L. 3.85 = ſum in hand, or preſent value.

$\frac{M - \overline{NM}}{N} =$ L. 0.2622 = annual rate, excluſive of the expence of management, payable during the life of A, to ſecure L. 1 annuity to B, after A's deceaſe, or, failing B, 10 years purchaſe of her annuity to their children.

This, I acknowledge, is different from the uſual method of conſtructing a widows ſcheme, and in particular from that recommended by Dr Price, whereby, to find the annual rate, the preſent value of the annuity muſt be divided by that of the joint lives, as in Caſe 1ſt; making the full value of the annuity, in the preſent inſtance, to be paid in 19, whereas A hath a chance of living 28 years: Beſides,

L. 3.85 val.
L. .2622 ann.
L. .3357 ann. } accumulated during A's life, by Tab. IV. = { L. 13.74
L. 16.83
L. 21.55 and

val. of B's ann. at widowity, per ſaid tab. = L. 14.20

The above rate or value, multiplied by any annuity

annuity, will, with the addition of the expence of management, ſhew the rate or value correſponding to that annuity. the ages being 30 and 25. Thus,

Exp.		*Ann. Rate.*	*Value.*	*Exp.*
.25 +		10 = 2.872	L. 38.5	L. 25 p. an.
.375 +		15 = 4.308	57.75	.375
.5 +	.2622 ×	20 = 5.744	77.0 with	.5
.625 +		25 = 7.180	96.25	.625
.75 +		30 = 8.616	115.5	.75

Again, let the reſpective ages of A and B be 40 and 30 years, the annuity L. 20, and the rate of intereſt 4 per cent; then $4.256 \times 20 = L.85.12$ = value of the annuity; and, $.5 + .3225 \times 20$ = L. 6.95 = annual rate. But the number at the former ages may be to that at the latter, as 3 : 1; thence $\frac{5.744 \times 3 + 6.95}{4} =$ L. 6.05, the mean annual rate correſponding to L. 20 annuity.

Had theſe rates been conſtructed upon the principles of the expectation of life, as in the Church's Scheme; in this caſe, the preſent value of an annuity for 20 years, at 4 per cent. = 13.590

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divided by the amount thereof for 30 years = 56.085 quots the annual rate = L..2423, which is leſs than the above. In ſuch a ſcheme, in order to aſſiſt their operation, and to balance the advantage of the marriage-tax in the Church's Scheme, the rates might be paid per advance;—the widow's annuity may begin to run from the firſt term of Whitſunday or Martinmas after the deceaſe of the contributor;—where there is no widow, 10 years purchaſe of her annuity might be granted to the children of a contributor, with-

in a twelvemonth after his deceaſe;—if a contributor, before his death, hath not paid 10 annual rates, a deduction might be made out of the proviſion of the widow or children, till the balance be recovered;—and for each 500 contributors, the annual expence of management ought not to exceed L. 110.

Suppoſing then, that there were 500 married contributors in this ſcheme, of 30 years of age and upwards, whoſe number being conſtantly the ſame,

Their expectation of life = 32.5 years
The number dying yearly = 15.0

of whom, 11 will leave widows, 3 of them, families of children without a widow, and 1, neither a widow nor children.

The annual number of marriage taxes, each at L. 9, = 3.0
The expectation of widowity, = 19.5
The maximum of annuitants, = 214.0

of whom 203 will draw full, and 11, including heirs, half annuities; and the families of children, = 3.0

Deductions out of the proviſion of widows or children, = L. 82.6
The medium of annuities L. 20, and of rates, L. 6.05
The annual expence of management = L. 110
And the capital neceſſary to ſupport the ſcheme, when the annuitants ſhall arrive at a maximum, = L. 41,390

Of a Marriage-Tax.

70. A marriage-tax may be conſidered as the difference

ference in point of value, of the two Widows Annuities, computed at the commencement of the ſecond marriage: Thus,

Suppoſing the above contributor becomes a widower, and at the age of 40 marries a ſecond wife, whoſe age is 30 years; ſought, his marriage-tax, at 4 per cent.

From the value of two joint lives aged 40 and 30 years, = L. 4.256
Subtract that of two joint lives aged 40 and 35 years, = 3.822

The remainder, = .434
Multiplied by 20, will produce the tax for that annuity, = L. 8.68

Queſt. A and B, whoſe reſpective ages are 60 and 40 years, are ſubjected to the burdens and enjoy the advantages of a widows ſcheme, wherein B's annuity, ſhould ſhe ſurvive A, is L. 25; B, deſigning to ſell her annuity, deſires to know the preſent value thereof, reckoning intereſt at 4 per cent.

The expectation of the joint continuance of theſe two lives, = 10.55
And B's probability of ſurviving the diſſolution of marriage is of conſequence, $\frac{35.45}{46.00}$ = .77

But $M - \overline{NM} = 5.686$ would be the years purchaſe of the annuity, were it certain that B would be the ſuvivor. Hence,

$.77 \times 5.686 \times 25 =$ L. 109.455, is the value required.

Or thus, A's expectation of life = 13, and B's probability of ſurviving A = $\frac{m-n}{m} = \frac{33}{45}$ = .717;

hence, B's age at A's death would be 53 years, whoſe value, viz. 10.702, reduced for 13 years, at 4 per cent, gives 6.43, the years purchaſe of the annuity, were it certain that ſhe would ſurvive A; hence, $.717 \times 6.43 \times 25 = \text{L. } 115.25$, is the value required.

PROBLEM IX.

It is required, to find the value of the reverſion of an annuity, which is to continue during the joint lives of two perſons, after the deceaſe of the preſent poſſeſſor, at a given rate of compound intereſt.

71. Let x, y, and z be the reſpective probabilities of the continuance of the three lives for one year; then $1-z$ will be the probability of the poſſeſſor's life failing the firſt year; which, multiplied by xy, gives $xy-xyz$, the probability that the two expectants will outlive the poſſeſſor the firſt year. Hence, by ſubſtituting the values of the lives in place of the probabilities thereof,

$\overline{NM} - \overline{NMF}$ = the value of the reverſion.

Therefore, if, from the value of the two joint lives in expectation, the value of the three joint lives be ſubtracted, the remainder will be the reverſion of two joint lives after one.

Ex. Let the reſpective ages of the two expectants and poſſeſſor be 43, 54, and 66 years, and the rate of intereſt 4 per cent. then,

From

From the value of the two joint lives
in expectation = 8.364
Subtract that of the three joint lives = 5.152

The remainder = 3.212 is the value of the reverſion of two joint lives after one.

PROBLEM X.

To find the value of the reverſion of an annuity, which is to continue during one life, after two joint ones, at a given rate of compound intereſt.

72. Let xy repreſent the probability of the two lives in poſſeſſion outliving the firſt year; then $\overline{1-xy} \times z = z - xyz$ will be the probability of the life in expectation outliving the two lives in poſſeſſion the firſt year; hence,

$F - \overline{NMF}$ will be the value of the reverſion; Therefore,

If, from the value of the expectant's life, the value of the three joint lives be taken, the remainder will be the value of the reverſion of one life, after two joint lives.

Ex. Let the age of the life in expectation be 43 years, and the ages of the two lives in poſſeſſion 54 and 66 years, allowing compound intereſt at 4 per cent. Then,
From the value of the expectant's life = 12.683
Subtract the value of the three joint lives = 5.152

There remains the value of the reverſion = 7.531

PROBLEM

PROBLEM XI.

To find the value of the reverſion of an annuity, which is to continue during the life of a given age, after the longeſt of two lives, whoſe ages are alſo given, at a given rate of compound intereſt.

73. Let x, y, and z repreſent the reſpective probabilities of the three lives continuing one year. Then $z \times \overline{1-x} \times \overline{1-y} = z - xz - yz + xyz$ will be the probability of the life in expectation outliving the other two the firſt year; and $F - \overline{FM} - \overline{NF} + \overline{NMF}$ will be the value of the reverſion of the life in expectation after the other two.

But if the reſpective values of the longeſt of two and three lives, Prob. IV. and V. be compared with this, it will appear to be their difference; Therefore,

If, from the value of an annuity on the longeſt of three lives, the value of an annuity on the longeſt of the two in poſſeſſion be taken, the remainder will be the value of the reverſion on the third life, after the other two.

Ex. A, who is 43 years of age, is intitled to the reverſion of an eſtate for his life, after the deceaſe of his father and of his mother-in-law, whoſe reſpective ages are 66 and 54 years; ſought, the value of A's intereſt in the eſtate, allowing 4 per cent.

From

From the value of the longeſt of the three lives	=	15.190
Subtract the value of the longeſt of the two poſſeſſors lives	=	11.904
Remains the value of the reverſion,	=	3.286

PROBLEM XII.

To find the value of the reverſion of an annuity, which is to continue during the longeſt of two lives after one, at a given rate of compound intereſt.

74. Let $\overline{1-x} \times \overline{1-y} = 1-x-y+xy$ repreſent the probability of the two lives failing in one year, which, being ſubtracted from unity, leaves $x+y-xy$, the probability of one, at leaſt, of the two lives outliving the year; and this laſt expreſſion, being multiplied by $1-z$, gives $x+y-xy-xz-yz+xyz$, the probability that the longeſt of the two lives will ſurvive the third the firſt year. Hence $N+M-\overline{NM}-\overline{NF}-\overline{MF}+\overline{NMF}$ will be the value of the reverſion of an annuity upon the longeſt of two lives after one; but this value is the difference of the reſults of Prob. I. and V. Therefore,

If, from the longeſt of the three lives of a poſſor and two expectants, be taken the value of the poſſeſſor's life, the remainder will be the reverſion of the longeſt of two lives after one.

Ex. Let the reſpective ages of the two expectants be 43 and 54, and that of the poſſeſſor 66 years, and the rate 4 per cent.

From

From the value of the longeſt of the three lives =	15.190
Take the value of the poſſeſſor's life =	7.333
Remains the value of the reverſion =	7.857

PROBLEM XIII.

To find the value of the reverſion of an annuity, where a given term of years is concerned.

75. 1ſt, To find the value of the reverſion of an aſſigned life after a given term of years.

Ex. A, aged 15 years, expects to enter upon an eſtate of L. 100 per annum, after the expiration of 10 years, which he is to hold during life; ſought, the value of his expectation, reckoning intereſt at 4 per cent. Here,

From the value of the propoſed life =	16.410
Subtract the value of an annuity for the given term of years, on the contingency of its ceaſing upon the extinction of the propoſed life, found by Prob. VI. =	7.540
The remainder multiplied by 100 = L. 887, is the value of A's expectation,	8.870

2d, To find the value of the reverſion of an annuity, for the remainder of a given term of years after an aſſigned life.

Ex. A, aged 25 years, who has the right of an annuity

annuity for 31 years certain, makes over the reverſion thereof to B and his heirs, to enjoy the ſame after his deceaſe for the remainder of the ſaid term; ſought, the value of B's expectation, reckoning intereſt at 4 per cent.

From the value of the annuity certain for 31 years, =	17.588
Subtract the value of the annuity for the ſame term, on the contingency of its failing on the extinction of A's life, found by Prob. VI. =	13.858
Remains the value of B's expectation, =	3.730

3d, To find the value of the reverſion of an annuity for a certain number of years, after an aſſigned life.

Ex. Let the given age be 50, the term of years 12, and the rate of intereſt 4 per cent. Then,

The value of the life is 11.344, the neareſt to which in Tab. V. of annuities certain is 11.118, correſponding to 15 years;

But 12 + 15 = 27 = 16.496 the value of the life and reverſion;

Subtract the value of the life = 11.344

There remains, 5.152 = the value of the reverſion of 12 years, after a life of 50.

4th, A perſon, 35 years of age, wants to buy an annuity for what may happen to remain of his life after 50 years of age; ſought the value of ſuch an annuity in ready money, and alſo in annual payments for 15 years, till he attains to

the ſaid age, ſubject in the mean time to failure, ſhould his life fail.

The preſent value of ſuch an annuity is equal to the value of a life at 50, multiplied by the preſent value of L. 1 to be received at the end of 15 years, and alſo by the probability that the given life will continue ſo long. That is,

$11.344 \times .5553 \times \frac{346}{490} = 4.44$, the number of years purchaſe that ought to be given for the annuity, reckoning intereſt at 4 per cent.

Now, ſuppoſing the annuity L. 50, its value in preſent money is L. 222,

Or thus: From the value of a life of 35 years, =	13.979
Subtract its value for 15 years, ſubject to failure, =	9.533
Remains the years purchaſe of the annuity, =	4.446

And L. 222, the value of the annuity in ready money, divided by 9.53, its value for 15 years, gives L. 23.3, the annual payments till the life attains to 50 years.

5th, To find the preſent value of an annuity to be enjoyed by one life, for what may happen to remain of it beyond another life of equal age, after a given term; that is, provided both lives continue to the end of a given term of years. Here,

Multiply the value of the annuity for the two joint lives, plus the given term of years, diſcounted for the given term, by the probability that the two given lives ſhall both continue the given term; the product will be the anſwer.

Ex.

Ex. Let the two lives be each 30, the term 7 years, the annuity L. 10, and the intereſt 4 per cent.

From the value of a ſingle life at 30+7 = 13.67
Subtract the value of two joint lives, each 37 = 10.25
Remains, the value of an annuity for the life of a perſon of 37 years of age, after another of the ſame age, which, being diſcounted for 7 years, = 2.6. } 3.42

The probability that two lives at 30 ſhall continue 7 years is (by Mr De Moivre's hypotheſis) $\frac{49}{56} \times \frac{49}{56} = .765$; and $2.6 \times .765 = 1.989$, the number of years purchaſe which ought to be given for an annuity, to be enjoyed by a life now 30, after a life of the ſame age, provided both continue 7 years; and the annuity being L. 10, its preſent value is L. 19.89.

In like manner, ſuppoſing the term one year, and the ages and the rate of intereſt the ſame, the preſent value of the ſame reverſionary annuity is L. 32.4; and if the term is 15 years, the value is L. 9.7.

Hence, ſuppoſing the ſcheme of a ſociety for granting annuities to widows to be, that if a member lives 1, 7, or 15 years after admiſſion, his widow, of the ſame age with himſelf, ſhall be intitled to an annuity of L. 20, L. 30, or L. 40; we may find what ought to be the annual payment of each member of a given age, and at a given rate of compound intereſt.

By the above, the value of L. 10 per annum, to a life of 30 after another of 30, provided the joint lives do not fail in one year, is L. 32.4, at 4 per cent.

The value of L. 20 per annum, in the ſame circumſtances, is, therefore, =	64.80
The value of L. 10, after 7 years, is =	19.89
And of L. 10, after 15 years, is =	9.70
Their ſum =	94.39

is the value of the expectation in a ſingle preſent payment; which, being divided by 11.182, (the value of two joint lives at 30), gives L. 8.44, the annual payment during the joint lives. In like manner, were the ages 40 and 50, the annual payments would be L. 8.69 and L. 9.05.

PROBLEM XIV.

To find the value of an annuity, or the reverſion thereof, depending on ſurvivorſhip, and the expectation of life.

76. 1ſt, A and B, whoſe reſpective ages are 25 and 40 years, enjoy an annuity equally betwixt them; which, after the deceaſe of either of them, is to belong to the ſurvivor for life; ſought, the value of the right of each in that annuity, reckoning intereſt at 4 per cent.

From the value of A's life =	15.318
Subtract half the value of the joint lives =	5.327
Remains, the value of the right of A =	9.991
In like manner B's right =	7.869

2d, An annuity, after the deceaſe of A, aged 66, is to be divided equally between B and C, whoſe reſpective ages are 43 and 54 years, during their joint

joint lives, and then is to go entirely to the laſt ſurvivor for life; ſought, the value of B's expectation, reckoning intereſt at 4 per cent.

From the value of the reverſion of the life B after the life A, found by Problem VIII. =	6.433
Subtract half the value of the reverſion of the joint lives of B and C, after the life A, Problem IX. =	1.582
Remains, the value of B's expectation =	4.851

3d, A, aged 40, expects to come to the poſſeſſion of an eſtate, ſhould he ſurvive B, aged likewiſe 40, and offers to give ſecurity for L. 40 per annum out of the eſtate at his death, provided he ſhould get into poſſeſſion: What ſum ought now to be advanced to him, in conſideration of ſuch ſecurity, reckoning intereſt at 4 per cent.?

From the value of the perpetuity =	25.000
Subtract the value of the longeſt of two lives, each 40 =	16.566
Remains, the value of the reverſion of ditto =	8.434

which, multiplied by 40, gives L. 337.36, the value of the given eſtate, were it certainly to be enjoyed, after the extinction of the longeſt of two lives, both 40; but that A's life, in particular, ſhould fail laſt, rather than B's, is an even chance: The true value of the reverſion, therefore, is half the laſt value, or L. 168.68.

4th, B, aged 40, will, if he lives till the deceaſe of A, whoſe age is 30, become poſſeſſed of an eſtate of L. 40 a-year; ſought, the worth of his

his expectation in preſent money, reckoning intereſt at 4 per cent. Here aſſume two lives, whoſe common age is equal to that of the older of the two propoſed lives, A and B, and find the expectations of life in A and B, viz. 28 and 23. Then,

From the value of the perpetuity =	25.000
Subtract the value of two joint lives, each 40 =	9.826
Remains, the reverſion of the two joint lives =	15.174

the half of which, viz. 7.587 would be the value of B's expectation, were their lives each 40: Therefore ſay,

As 28 : 23 : : 7.587 : 6.232, which, multiplied by 40, gives L. 249.28, the value of B's expectation.

If B's age be 30, and that of A 40 years; then,

From the value of the perpetuity =	25.00
Subtract the value of the joint lives B and A, viz. 10.428 + 6.232, found above, =	16.66
Remains, the value of B's expectation =	8.34

5th, C and his heirs are intitled to an eſtate of L. 200 per annum, upon the deceaſe of B, aged 40, provided B ſurvives A, whoſe age is 30 years; ſought, the value of their expectation in preſent money, reckoning intereſt at 4 per cent. Aſſume two equal lives, whoſe common age is that of the older of the two propoſed lives, B and A; then,

From

From the value of the perpetuity =	25.000
Subtract that of the longeſt of two equal lives, each 40 =	16.566
Remains, the reverſion of ditto =	8.434

the half of which, viz. 4.217, would be the years purchaſe required, if the ages of A and B were each 40 years: Therefore,

28 : 23 :: 4.217 : 3.464, the number of years purchaſe required, anſwering to L. 692.8.

If B's age had been 30, and A's 40 years, in this caſe,

From the value of the perpetuity =	25.000
Subtract that of the longeſt of the two lives B and A, viz. 17.452+3.464, found above =	20.916
Remains, the value of C's expectation =	4.084

Hence, in a widows ſcheme, when the reſpective ages of huſband and wife are 30 and 25 years, and the proviſion for a family of children, where there is no widow, L. 200; we may find the preſent value of ſaid proviſion, reckoning intereſt at 4 per cent.

By the above, the years purchaſe of the children's proviſion would be 3.128; but the intereſt of L. 200, for one year, at 4 per cent. is L. 8: Therefore 3.128 × 8 = L. 25.024, is the expectation of a family of children, without a widow.

PRO-

PROBLEM XV.

Of Inſurance on Lives.

77. 1ſt, IF A, aged 30 years, is willing to advance money in hand, or to pay a certain ſum yearly during life, to ſecure L. 100 to his heirs; or, in other words, to inſure his life upon L. 100, allowing intereſt at 4 per cent. In this caſe, $N=14.684$, and (Problem VII. cor. 1.) $\frac{P}{n}=40\times 100=$L. 40, the ſum payable in hand; which, divided by 14.684, gives L. 2.73, the annual payments during life.

2d, A, aged 40 years, agrees to advance a certain ſum in hand, or to make annual payments during the ſpace of 15 years, in caſe he lives ſo long, in order to ſecure L. 300 to him or his heirs; ſought, the preſent value of ſaid ſum, or annual payments during theſe 15 years, reckoning intereſt at 4 per cent. Here, (Problem VI.), $N=8.817$, and $\frac{1-rN}{R}=.6224\times 300=$L. 186.7, the preſent value; which, divided by 8.817, gives L. 21.20, the annual payments during 15 years, in caſe A lives ſo long.

3d, A, aged 40 years, wants to make proviſion for B, aged 30, in caſe the latter ſhould happen to ſurvive: What ought the former to give in a ſingle payment, and alſo in annual payments during their joint lives, to ſecure L. 500, payable at his death to the latter, reckoning intereſt at 4 per cent.?

In

In this caſe, the value of the joint lives of A and B = 10.428, L. 500, at 4 per cent, is equal to an eſtate of L. 20 per annum, and (Problem XIV. 4.) the value of B's expectation of ſurviving A is 8.34; therefore, 8.34 × 20 = L. 166.8, is the ſingle payment, and 166.8, divided by 10.428, gives L. 16, the annual payments during the joint lives of A and B.

4th, An eſtate of L. 20 per annum will be loſt to the heirs of a perſon now 40, ſhould his life fail in 12 years: What ought he to give for the aſſurance of it for that term, reckoning intereſt at 4 per cent.? Or, What is the preſent value of the eſtate, to be entered upon at the failure of ſuch a life, ſhould that happen in 12 years?

From the value of an annuity certain for 12 years =	9.385
Subtract the value of the life ſubject to failure in 12 years =	8.154
Remains, the value of the eſtate for 12 years, upon the chance of the life's failing in that time.	1.231

Again, 25, the perpetuity, × .6246, the value of L. 1, diſcounted for 12 years × $\frac{121}{445}$, the chance of the life's failing in that time, gives 4.246, the value of the eſtate after 12 years, upon the chance of the life's failing in that time; and theſe two values, added together, give 5.477, the value of inſurance, correſponding to L. 109.54, which, divided by 8.154, the value of the life, quots L. 13.43, the annual payments.

5th, There is an eſtate, which, if A, aged 7 years, happens to die before he attains to the age of 21, is, after his deceaſe, to go to B and his heirs for ever; ſought, the value of B's expectation, reckoning intereſt at 4 per cent.

If the reverſion of the eſtate, after A's life had belonged to B and his heirs; then, in this caſe,

From the perpetuity =	25.000
Subtract the value of a life 7 years old =	16.698
There would remain B's expectation =	8.302

But B's expectation is limited to 14 years of A's life; therefore,

From the perpetuity =	25.000
Subtract the value of a life 21 years old =	15.781
Remains, the reverſion of a life of that age =	9.219

and $9.219 \times .5775$, the value of L. 1, diſcounted for 14 years $\times \frac{592}{692}$, the chance of A's living 14 years, gives 4.554; which, being ſubtracted from the former value, leaves 3.748, the value of B's expectation.

Again, had the reverſion of the eſtate been limited to B, aged, for inſtance, 30 years; in this caſe, $3.748 \times \frac{407}{531}$, the chance of B's living 14 years, gives 2.873, the value of B's expectation; which, multiplied by 4, the intereſt of L. 100 for one year, produces L. 11.6, the preſent value of L. 100 to be paid by B, in caſe the ſurvivorſhip ſhould take place.

PRO-

PROBLEM XVI.

To find the Value of Succeſſive Lives.

78. LET N and M repreſent the values of an annuity of L. 1, to continue n and m years, at a given rate of compound intereſt;

then $N=\frac{1-\frac{1}{R^n}}{r}$ (47)

and $M=\frac{1-\frac{1}{R^m}}{r}$

which gives $\frac{1}{R^n}=1-rN$, $\frac{1}{R^m}=1-rM$.

Hence, $\frac{1}{R^{n+m}}=\overline{1-rN}\times\overline{1-rM}=1-rN-rM+r^2NM$. Now,

let V be the value of an annuity, which is to continue $n+m$ years; then, $\frac{1}{R^{n+m}}=1-rV=\overline{1-rN}\times\overline{1-rM}$; which gives,

$V=\frac{1-\overline{1-rN}\times\overline{1-rM}}{r}=N+M-rNM$, the value of an annuity to continue during two ſucceſſive lives, whoſe ſingle values are N and M.

Ex. 1*ſt*, A, aged 40 years, enjoys an annuity for his life; and, at his deceaſe, has the nomination of a ſucceſſor, aged 18 years, who is alſo to enjoy the annuity during his life; ſought, the

preſent value of the two ſucceſſive lives, reckoning intereſt at 4 per cent. Here $N=13.196$, and $M=16.105$.

Hence, $N+M-rNM=29.301-8.5=20.810$, the value required.

In like manner, if there be three ſucceſſive lives, whoſe ſingle values are N, M, and F, then,

$V=\frac{1-\overline{1-rN}\times\overline{1-rM}\times\overline{1-rF}}{r}$ the value of three ſucceſſive lives.

Again, if the two ſucceſſive lives are of the ſame age, which is moſt likely, then $n+m=2n$, and

$$\frac{1}{R^{2n}}=1-rV=\overline{1-rN}^{2}, \text{ which gives}$$

$V=\frac{1-\overline{1-rN}^{2}}{r}$ the value of two equal ſucceſſive lives,

and $V=\frac{1-\overline{1-rN}^{3}}{r}$ the value of three equal ſucceſſive lives.

Ex. 2d. It is required, to find the value of four equal lives, following one another in ſucceſſion, whoſe common age, at admiſſion, is 24 years, reckoning intereſt at 4 per cent. Here $N=15.437$ and

$$\frac{1-\overline{1-rN}^{4}}{r}=\frac{.9786}{.04}=24.465, \text{ the value.}$$

Univerſally, if N ſtands for the value of an annuity to continue a certain number of years, then

then $\frac{1-\overline{1-rN}^{n}}{r}$ will represent the value of an annuity to continue n times as long. If n is indefinite, then $\overline{1-rN}^{n}=o$, and $v=\frac{1}{r}$, the value of the perpetuity.

PROBLEM XVII.

Of a Copy-hold, and the renewing of Leases.

79. SUPPOSING any number of equal lives of a given age, and that upon the failing of any one, or all of them, they shall be immediately replaced, and I shall then receive a sum of money S, agreed upon; sought, the value of that expectation, and at what intervals of time I may expect to receive the said sum.

Let n denote the number of years during the intervals betwixt the payments of the sum S; and let N represent the value of the life or lives during these intervals; then,

$$N=\frac{1-\frac{1}{R^{n}}}{r}\ (47.)\ \text{which gives}\ R^{n}=\frac{1}{1-rN}.$$

Now seeing a sum of money, S, is to be received, *ad infinitum*, at the equal intervals of time denoted by n; its present value may be found from the following series, viz. $\frac{S}{R^{n}}+\frac{S}{R^{2n}}+\frac{S}{R^{3n}}$, &c. $=\frac{S}{R^{n}-1}$ (40.) the present value of all the payments.

But

But in place of R^n, ſubſtitute its value found above, viz. $\frac{1}{1-rN}$,

then $R^n-1=\frac{rN}{1-rN}$, and conſequently

$\frac{S}{R^n-1}=\frac{1-rN}{rN}\times S$, the value required.

Again, let $\frac{1}{1-rN}=Q$, then $R^n=Q$, and

$n=\frac{Log.\ Q}{Log.\ R}$ the intervals of time at which I may expect the ſum S.

Ex. 1ſt. Let there be three perſons, each 20 years of age, at the deceaſe of any one of which, I ſhall receive L. 250, which life is to be immediately replaced, and whenever a life becomes vacant, I or my heirs are to receive a like ſum; ſought, the value of that expectation, and at what equal intervals of time I may expect the ſaid ſum, allowing intereſt at 4 per cent.

Here, $N=9.192$ by Prob. III. and

$\frac{1-rN}{rN}\times S=1.72\times 250=$L. 409, the value required.

Again, $n=\frac{.1990631}{.0170333}=11.68$ years of interval.

Ex. 2d. A purchaſes a leaſe-hold eſtate, upon three equal lives, aged 15 years, for the ſum of L. 2000, on condition that his heirs fill up the leaſe continually, whenever all the lives become vacant, paying a fine of L. 1000; ſought, the preſent value of the whole purchaſe allowed for the eſtate, with the annualrent anſwering thereto, and at what equal intervals of time the heirs of

A

A may expect to pay the ſaid fine, allowing intereſt at 4 per cent.

Here $N=20.25$, by Prob. III. and $\frac{1-rN}{rN}\times s=.2346\times 1000=$L. 234.6 the value of the fines;

To which add, 2000.0

Their ſum is, 2234.6 the value of the purchaſe, whoſe annual rent, at 4 per cent. is L. 89.384; and

$n=\frac{.7212464}{.0170333}=42.3$ years of interval.

If only one life is concerned, which, at the demiſe of the incumbent, is to be replaced with a ſucceſſor, as in the caſe of a benefice, or of a copy-hold; in which, let N denote the value of the life, and s the fine, if there is any, to be paid on admiſſion; then,

$\frac{1-rN}{rN}\times s=$ value of the copy-hold or right of patronage, to which add the ſum s, if the purchaſe is made during the vacancy; then $\frac{s}{rN}=$ the value of the patronage.

Ex. 3d, Let an intrant's age be 30 years, the fine on admiſſion L. 150; ſought, the value of the patronage at 4 per cent. Here $N=14.684$, and $s=150$. Hence,

$\frac{1-rN}{rN}\times s=.7025\times 150=$L. 105.375, the value ſought. Or, if the purchaſe be made during the vacancy, then,

$\frac{s}{rN}=$L. 255.375, the value of the patronage.

PRO-

PROBLEM XVIII.

Of the Expectation of Life.

80. THE expectation of life is that period of time, during which a perſon of a given age may juſtly expect to continue in being, and is always equal to the years purchaſe of an annuity of L. 1 during life, upon the ſuppoſition that money bears no intereſt.

Caſe 1ſt, It is required, to find the expectation of a ſingle life of a given age, ſuppoſing the decrements of life to be equal.

In eſtimating the expectation of life, we muſt reckon from the preſent inſtant, and ſuppoſing n to be the complement of life, make $\frac{n}{n} = 1$, the firſt term of the ſeries, $\frac{n}{n} + \frac{n-1}{n} + \frac{n-2}{n}$, &c. expreſſing the expectation of life; which being a ſeries decreaſing every inſtant, the laſt term thereof viz. $\frac{n-n}{n} = 0$, and the number of terms, though indefinite, is expreſſed by n; therefore, $\frac{n}{n} \times \frac{n}{2} = \frac{n}{2}$ (8) the ſum of the ſeries is equal to the expectation of that ſingle life whoſe complement is n.

Ex. A perſon of 24 years of age, may expect to live $\frac{n}{2} = \frac{62}{2} = 31$ years longer.

2d,

2d, It is required, to find the expectation of two joint lives, of given ages.

Let the complement of the elder life be n, and that of the younger m; then the ſum of the ſeries $\frac{n}{n} \times \frac{m}{m} + \frac{n-1}{n} \times \frac{m-1}{m} + \frac{n-2}{n} \times \frac{m-2}{m} (n)$ will be the expectation of the joint lives. Now, the terms of this ſeries are the products of the two arithmetical progreſſions, which expreſs the expectations of the ſingle lives, carried on to a number of terms equal to n; but the ſum of the firſt is $\frac{n}{2}$, and, ſince the greateſt term of the ſecond is $\frac{m}{m}$, the leaſt $\frac{m-n}{m}$, and the number of terms n, the ſum thereof will be

$\frac{m}{m} + \frac{m-n}{m} \times \frac{n}{2} = \frac{2m-n}{m} \times \frac{n}{2} = n - \frac{nn}{2m}$; (7, Equ. 3d.) hence, the ſum of n terms of the ſeries of products will be $=$

$\frac{\frac{n}{2} \times n - \frac{nn}{2m} \times n}{n} + \frac{n+1 \times n \times n-1}{2.2.3} \times \frac{1}{n} \times \frac{1}{m}$, which being reduced, becomes $\frac{n}{2} - \frac{nn}{4m} + \frac{nn-1}{12m}$, and, neglecting $\frac{1}{12m}$, it will become $\frac{n}{2} - \frac{nn}{6m}$, the expectation required.

Ex. Two perſons, whoſe reſpective ages are 30 and 25 years, may expect to live together $28 - 8.568 = 19.43$ years.

If the two ages are equal, the expectation of their joint lives will be $\frac{n}{2} - \frac{nn}{6n} = \frac{n}{2} - \frac{n}{6} = \frac{n}{3}$.

3d, It is required, to find the expectation of any number of equal joint lives, of a given age.

Let there be, for inſtance, two equal lives;

then the firſt term of the ſeries, expreſſing the expectation of their joint continuance, will be $\frac{n}{n} \times \frac{n}{n} = 1^2$; and the other terms will alſo be ſquares, ſo that the whole will be a ſeries of ſquares (whoſe roots are in arithmetical progreſſion) beginning with 1, and ending in a cypher, and n the number of terms; therefore, calling m the index of the power in the ſeveral terms, $\frac{n \times 1}{m+1} = \frac{n}{3}$ will be the ſum of the ſeries, or expectation required: See the following note *. Or let there be three equal lives: then the firſt term of the ſeries expreſſing the expectation of their joint continuance will be $\frac{n}{n} \times \frac{n}{n} \times \frac{n}{n} = 1^3$; and $\frac{n \times 1}{m+1} = \frac{n}{4}$, their expectation required; and univerſally calling any number of equal lives m, and their complement of life n, then $\frac{n}{m+1}$ will expreſs the expectation of their joint continuance in being; ſee the following note *.

Ex.

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* The ARITHMETIC of INFINITIES.

If there be any ſeries of quantities, all of the ſame power, whoſe roots are in arithmetical progreſſion, 0, 1, 2, 3, 4, &c. and let m denote the index of the power; N the number of terms; L the laſt term; and S the ſum of the ſeries; then, $\frac{NL}{m+1} = S$. Thus,

$$\left.\begin{array}{lllll} 0, & 1, & 2, & 3, & 4, \\ 0^2, & 1^2, & 2^2, & 3^2, & 4^2, \\ 0^3, & 1^3, & 2^3, & 3^3, & 4^3, \\ 0^4, & 1^4, & 2^4, & 3^4, & 4^4, \\ \sqrt{0}, & \sqrt{1}, & \sqrt{2}, & \sqrt{3}, & \\ \sqrt[3]{0}, & \sqrt[3]{1}, & \sqrt[3]{2}, & \sqrt[3]{3}, & \end{array}\right\} m = \left\{\begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ \frac{1}{2} \\ \frac{1}{3} \end{array}\right\} m+1 = \left\{\begin{array}{l} 2 \\ 3 \\ 4 \\ 5 \\ \frac{3}{2} \\ \frac{4}{3} \end{array}\right\} \frac{NL}{m+1} = \left\{\begin{array}{l} \frac{NL}{2} \\ \frac{NL}{3} \\ \frac{NL}{4} \\ \frac{NL}{5} \\ \frac{2NL}{3} \\ \frac{3NL}{4} \end{array}\right\} = S$$

This

Ex. Fifty perſons, whoſe age at a medium is 30 years, may expect to continue all in being $\frac{56}{51} = 1$ year 36 days.

4th, It is required to find the expectation of three joint lives, whoſe ages are given. Let the complements of the eldeſt, ſecond, and youngeſt, be ſeverally denoted by n, m, and t; then by following a proceſs ſimilar to that obſerved in caſe 2d, the expectation of their joint continuance would be found to be,

$\frac{n}{2} - \frac{n^2}{6} \times \overline{\frac{1}{m} + \frac{1}{t}} + \frac{n^3}{12mt}$. See the note in caſe 9th.

Ex. Three perſons, whoſe reſpective ages are 50, 45, and 35 years, may expect to continue all in being, $18 - 9.59 + 1.86 = 10.27$ years.

5th, It is required, to find the expectation of the longeſt of two lives, of given ages. Here, by reaſoning as in Prob. IV, and calling N and M the expectations of the ſingle lives, and $\overline{NM}$ that of their joint continuance; then,
$N + M - \overline{NM}$ will be the expectation of the longeſt of theſe two lives.

Ex. Sought, the expectation of the longeſt of

This may be proven by induction, whereby it is ſhewn, that the greater the number of terms is, the nearer will the expreſſion $\frac{NL}{m+1}$ approach to the ſum of the ſeries, and is only equal to it when the number of the terms is indefinite.

two lives, whoſe reſpective ages are 30 and 25 years. Here,
$28+30.5-19.43=39$, the expectation of the longeſt liver.

6th, It is required, to find the expectation of the longeſt of three lives of given ages. Let N, M, and F repreſent the expectations of the three ſingle lives, and $\overline{NMF}$ that of their joint continuance; then, by reaſoning as in Prob. V. $N+M+F-\overline{NM}-\overline{NF}-\overline{MF}+\overline{NMF}$ will be the expectation of the longeſt of theſe three lives.

7th, If the ages are equal, then the ſum of the ſeries $1-\frac{\overline{o}^{m}}{n}+1-\frac{\overline{1}^{m}}{n}$, &c. will be the expectation of the longeſt liver: but this ſeries is equal to a number of units whoſe ſum is n, minus a ſeries of m powers, whoſe roots are in arithmetical progreſſion, beginning with 1, and ending in 0; therefore $n-\frac{n}{m+1}=\frac{mn}{m+1}$ is the ſum of the ſeries. Hence,

$\left|\begin{matrix}\frac{2n}{3}\\ \frac{3n}{4}\\ \frac{4n}{5}\\ \frac{mn}{m+1}\end{matrix}\right|$ = The expect. of the longeſt of $\left|\begin{matrix}2\\ 3\\ 4\\ m\end{matrix}\right|$ equal lives whoſe complement of life is n.

Ex. Let there be 100 perſons, whoſe age, at a medium, is 30 years, ſome of them may expect to live $\frac{5600}{101}=55.44$ years.

8th,

8th, It is required, to find the expectation of widowhood in married perſons, whoſe ages are given.

From the expectation of the longeſt liver $= N + M - \overline{NM}$
Subtract that of their joint continuance $= + \overline{NM}$

Remains the expectation of widowhood $= N + M - 2\overline{NM}$

Ex. In married perſons whoſe reſpective ages are 30 and 25 years, the expectation of widowhood is $58.5 - 38.86 = 19.64$.

In married perſons of equal ages, the expectation of widowhood is equal to that of marriage, ſeeing $\frac{2n}{3} - \frac{n}{3} = \frac{n}{3}$, which expreſſes either of them.

9th, If the expectation of life, for any number of years not exceeding n the complement thereof, be required; in this caſe calling x the number of years, the ſum of the ſeries $\frac{n}{n} + \frac{n-1}{n} \ldots + \frac{n-x}{n}$ will expreſs this expectation; but the ſum of the firſt and laſt terms $= \frac{2n-x}{n}$, and x the number of terms; hence, $\frac{2n-x}{2n} \times x$ (7. Equ. 3.) is the expectation required. When $x = n$, the expreſſion becomes $\frac{n}{2}$, and gives the expectation of the aſſigned life for its whole poſſible duration.

Ex. The expectation of life for 15 years, in a perſon aged 30, is $\frac{97}{112} \times 15 = 12.99$ years.

Here it may be obſerved, that the above expreſſion alſo diſcovers the number of perſons alive, to which one perſon added annually to a ſociety, or left annually in widowhood, at a given age, will increaſe

increaſe in x years. Thus, calling $N=\frac{n}{2}$ the expectation of a widow's life; and ſuppoſing one to come upon a ſociety every year, the number of annuitants alive, deduced from hence, will, in x years, be $\frac{4N-x}{4N}\times x$. And calling W the number of widows left annually, at the ſame age, then $\frac{4N-x}{4N}\times Wx$ will be the number of annuitants alive at the end of x years. The maximum, therefore, is WN, which will always happen when $x=n$, the complement of the widow's life.

Ex. Let there come yearly upon a fund, 19.3 widows, whoſe expectation of life is 19.64, (caſe 8th;) it is required, to find the number to which they will increaſe in 15 years; and what will be their maximum. Here,

$$x=15,\ W=19.3,\ N=19.64, \text{ and,}$$

$\frac{4N-x}{4N}\times Wx=\frac{63.56}{78.56}\times 289.5=234.2$, the number of widows at the end of 15 years. And their maximum is $WN=379$. In like manner, the ſum of the ſeries $\frac{n^2}{n^2}+\frac{\overline{n-1}^2}{n^2}\cdots+\frac{\overline{n-x}^2}{n^2}=\frac{n^3}{3n^2}-\frac{\overline{n-x}^3}{3n^2}=x-\frac{x^2}{n}+\frac{x^3}{3n^2}$ not only expreſſes the expectation of two equal joint lives, for the time x, but alſo gives the number of marriages in being together, that will, in x years, grow out of one yearly marriage between perſons of equal ages, whoſe complement of life is n, which expreſſion, when $x=n$, becomes $\frac{n}{3}$, the expectation of two equal joint lives; or, calling M the number of yearly marriages, their maximum

maximum will be $\frac{nM}{3}$, at the end of n years. If the perſons are of unequal ages, let the complement of the oldeſt life be n, and that of the youngeſt m, the expreſſion will be $x-\frac{\overline{n+m}\times x^2}{2nm}+\frac{x^3}{3nm}$; and the number of marriages will come to a maximum when $x=n$.

Ex. Let there be a ſociety, wherein there are 32 yearly marriages, between perſons whoſe reſpective ages are 33 and 25 years; it is required, to find the greateſt poſſible number of married perſons, as alſo the maximum of widows and widowers in ſuch a ſociety. Here, $x=n=53$, $m=61$ and $M=32$; hence, $53-49.5+3.5=19$; which, multiplied by 32, gives 608, the greateſt number of married perſons; which number being equal to all the widows and widowers in life at one time, and the proportion of theſe to one another being as 5 to 3; hence, 8 : 5 :: 608 : 380, the maximum of widows, and 228, the greateſt number of widowers.

The EXPECTATION *of* LIFE *diſcovered by* FLUXIONS.

1*ſt*, Let $\frac{n-x}{n}$ repreſent the probability that a life, whoſe complement is n, will continue to the end of x time. Then $\frac{\dot{n}x-\dot{x}x}{n}$ will be the fluxion of the ſum of the probabilities, whoſe fluent, (to find which, ſeeing the fluxion of x^2 is $\dot{x}x+x\dot{x}=2x\dot{x}$, divide the whole by $\dot{x}$, add unity to the exponent of the power of x in each term, and divide by the exponent ſo increaſed), viz. $x-\frac{x^2}{2n}=\frac{2n-x}{2n}\times x$, is the ſum itſelf for the time x; this, when $x=n$, becomes $\frac{n}{2}$, and gives the expectation of the aſſigned life for its whole poſſible duration.

2*d*, Let

Supplement to Prob. XVIII.

Let N, M, P, and T repreſent the reſpective expectations of 4 lives, whoſe complements are n, m, p, and t; and c, the arithmetical mean complement

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2*d*, Let $\frac{\overline{n-x} \times \overline{m-x}}{nm}$ be the probability that two unequal joint lives will continue to the end of x time; $\frac{\overline{n-x} \times \overline{m-x}}{nm} \times \dot{x}$, will be the fluxion of the ſum of the probabilities; the fluent of this expreſſion, viz. $x - \frac{\overline{n+m}\,x^2}{2nm} + \frac{x^3}{3nm}$, when $x = n$, becomes $\frac{n}{2} - \frac{n^2}{6m}$, the expectation of two unequal joint lives, whoſe complements are n and m.

3*d*, Since $\frac{\overline{n-x}^2}{n^2}$ is the probability that two equal joint lives will continue to the end of x time, $\frac{\overline{n-x}^2}{n^2} \times \dot{x}$ will be the fluxion of the ſum of the probabilities, whoſe fluent, viz. $x - \frac{x^2}{n} + \frac{x^3}{3n^2}$, when $x = n$ becomes $\frac{n}{3}$ the expectation of two equal joint lives. Univerſally let the number of equal lives be m, then $\frac{n}{m+1}$ will be the expectation of their joint continuance.

4*th*, Let $\frac{\overline{n-x} \times \overline{m-x} \times \overline{t-x}}{nmt}$ be the probability that three unequal joint lives will continue to the end of x time; then this expreſſion, multiplied by $\dot{x}$, produces

$$\frac{\begin{array}{l} -mt\dot{x}x + tx^2\dot{x} \\ nmt\dot{x} - nt\dot{x}x + mx^2\dot{x} - x^3\dot{x} \\ -nm\dot{x}x + nx^2\dot{x} \end{array}}{nmt} = \text{the fluxion of the ſum of the probabilities;}$$

the fluent of which expreſſion, being reduced and x put $= n$, becomes $\frac{n}{2} - \frac{n^2}{6} \times \overline{\frac{1}{m} + \frac{1}{t}} + \frac{n^3}{12mt}$ = the expectation of three joint lives whoſe reſpective complements are, n, m, and t.

5*th*, Let

plement of all the lives wanting n, the complement of the oldeſt; then, $\frac{n}{3}$,

---

5*th*, Let $\frac{x}{n} \times \frac{x}{n} = \frac{x^2}{n^2}$ be the probability that two equal lives, whoſe complement is n, will fail in the time x; then $1 - \frac{x^2}{n^2} \times \dot{x}$ will be the fluxion of the ſum of the probabilities that one of them at leaſt will ſurvive the time x; the fluent of this expreſſion, viz. $x - \frac{x^3}{3n^2}$, when $x = n$, becomes $n - \frac{n}{3} = \frac{2n}{3}$, the expectation of the longeſt liver. Univerſally let the number of equal lives be m, then $\frac{mn}{m+1}$ is the expectation of the longeſt liver.

6*th*, Let $\frac{n-x}{n} \times \frac{2x}{n}$ be the probability that there will be a ſurvivor of two equal joint lives at the end of x time; then $\frac{2x}{n} - \frac{2x^2}{n^2} \times \dot{x}$ is the fluxion of the ſum of the probabilities, whoſe fluent, viz. $\frac{x^2}{n} - \frac{2x^3}{3n^2}$, when $x = n$, becomes $n - \frac{2n}{3} = \frac{n}{3}$, the expectation of ſurvivorſhip between two equal lives, and is the ſame with that of their joint continuance.

7*th*, Let $1 - \frac{x}{n} \times \frac{x}{m}$ be the probability that the oldeſt of two lives will ſurvive the other at the end of x time; then, $\frac{x}{m} - \frac{x^2}{nm} \times \dot{x}$ will be the fluxion of the ſum of the probabilities, whoſe fluent $\frac{x^2}{2m} - \frac{x^3}{3nm}$, when $x = n$, becomes $\frac{n^2}{6m}$. Hence the expectation of ſurvivorſhip on the part of the oldeſt $= \frac{n^2}{6m}$.

	Add	$\frac{m}{2} - \frac{n}{2}$
——— of the youngeſt	$=$	$\frac{m}{2} - \frac{n}{2} + \frac{n^2}{6m}$
	Add	$+ \frac{n^2}{6m}$
Expectation of widowity	$=$	$\frac{m}{2} - \frac{n}{2} + \frac{n^2}{3m}$

1*st*, The expectation of the joint continuance of 2 unequal lives $= \frac{n}{2} - \frac{n^2}{6m} = N - \frac{n}{3m} N = \overline{NM}$.

of 3 —— $= \frac{n}{2} - \frac{2n^2}{6c} + \frac{n^3}{12mp}$, $= \overline{NMP}$.

of 4* —— $= \frac{n}{2} - \frac{3n^2}{6c} + \frac{3n^3}{12c^2} - \frac{n^4}{20mpt} = \overline{NMPT}$.

of m equal lives $= \frac{n}{m+1}$

of m unequal lives $= \frac{2n}{m+1} - \frac{n^{m+1}}{m+1 \times nmpt}$

n being the mean comple. of all the lives.

Expectation of widowity, as on p. 201. $= \frac{m}{2} - \frac{n}{2} + \frac{n^2}{3m}$.

Add $\frac{n}{2} - \frac{n^2}{6m}$.

—— of the longeſt liver $= \frac{m}{2} + \frac{n^2}{6m}$.

8*th*, Let there be three joint lives, whoſe reſpective complements are n, m, and t; the expectation of ſurvivorſhip on the part of

the oldeſt $= \frac{n^2}{12mt}$.

the ſecond $= \frac{m^2}{12nt}$.

the youngeſt $= \frac{t^2}{12nm}$.

* To abridge the labour of calculating the expectation either of the joint continuance, or of the longeſt liver of m unequal lives, whoſe mean complement is n;

In the fluent $\frac{x^{m+1}}{m+1 \times nmpt}$, which expreſſes the ſum of the probabilities that all the lives will fail in the time, x; let $x = n$, the mean complement of all the lives, and ſubtracting that fluent from $\frac{2n}{m+1}$, we ſhall have $\frac{2n}{m+1} - \frac{n^{m+1}}{m+1 \times nmpt}$ = expectation of their joint continuance; and n — expectation of their joint continuance = that of the longeſt liver.

2d,

2d, The expectation of the longeſt liver

of 2 unequal lives $= N+M-\overline{NM} = M+\frac{n}{3m}N.$

of 3 $= \frac{n+m+p}{3} - \overline{NMP}.$

of 4 $= \frac{n+m+p+t}{4} - \overline{NMPT}.$

of m equal lives $= \frac{mn}{m+1}$

of m unequal lives $= n$—Expectation of their joint continuance | whoſe mean complement is n.

of q lives out of $m = \frac{qn}{m+1}$

Or in the caſe of 4, 5, 6 ... m unequal lives, find their mean age, and conſider it as the age of ſo many equal lives.

Ex. Sought, the expectation of the joint continuance and of the longeſt liver of three lives, whoſe reſpective complements are 40, 50, and 60. Here, $20-9.7\dot{7}+1.7\dot{7}=12$, Expect. of their joint contin.
$50-12 \quad =38$, ——— of the longeſt liver.

3d, If t, the time, be ſought, in which p, a part of m equal lives, will fail, whoſe complement is n; here,

$\frac{pn}{m+1} = t$, the time ſought.

$\frac{m+1}{n} \times t = p$; and making $t=1$,

$\frac{m+1}{n} = p$, the number which fail in one year; hence, In any number of lives, their expectation of life, and the number which die yearly, are reciprocal of, and diſcover one another.

4th, Let there be any conſiderable number of lives, in their natural order, extending from a

given age, whoſe complement is n, to the utmoſt period of human life; then,

1ſt, $\frac{2n}{3}$ = their mean complement of life.

The given age $+\frac{n}{3}$ = the mean age; and, $\frac{n}{3}$ = their mean expectation of life.

Thus in a table of equal decrements, where the complements of life form a ſeries of ſquares, $\frac{n^2}{2})\frac{n^2 \times n}{3} = \frac{2n}{3}$ the mean complement, &c.

Ex. 1ſt, In any approved table of obſervations, ſuch as that of Dr Halley, where 90 is the utmoſt extent of human life, 30 will be the mean age of all the perſons in the table, and its expectation, viz. 28, will be that of human life, or of infancy by the table.

Moreover, $28 \times 2 + 30 = 86$, is with propriety put as the utmoſt extent of human life by Mr De Moivre, in his Hypotheſis of equal decrements, which is adapted to Halley's table.

Ex. 2d, The marriages of miniſters wives, in the eſtabliſhed Church, take place from the age of 18 to 45 years; hence, $\frac{45-18}{3} + 18 = 27$, the mean age of marriage, which is alſo the mean age of the youngeſt widows; hence, $\frac{86-27}{3} + 27 = 46\frac{1}{3}$ the mean age of widowity, whoſe expectation of life, viz. $19\frac{2}{3}$, multiplied by 19.3, the number eſt yearly, will produce 379.5 the maximum of annuitants.

3d, In ſuch a number of lives, the expectation of

of the longeſt liver is nearly equal to the complement of life in the youngeſt.

4th, In ſuch a number of lives, which are conſtantly the ſame, being kept up by the addition of new lives as the old drop off, their expectation of life is the ſame with that of the youngeſt.

5th, To find the period of doubling the number of inhabitants in any province. When the number of births, with the addition of the annual ſettlers, does ſo far exceed the burials, that their difference bears a conſiderable proportion to the whole number of inhabitants, and is equal, ſuppoſe, to a 36th part thereof; the number of inhabitants, if this part continues conſtantly the ſame, will be doubled in 36 years. But, as every addition to the number of inhabitants, from the births and ſettlers, produces a proportionably greater number of births; if we ſuppoſe the exceſs to increaſe annually in proportion to the number of inhabitants, ſo as to preſerve the ratio of theſe to one another always the ſame; upon this ſuppoſition, the population of a province, till it doubles itſelf, will exactly reſemble the accumulation of money at compound intereſt: therefore, calling this ratio of the exceſs of the births and ſettlers above the burials, to the inhabitants, $\frac{1}{r}$;

$\frac{1}{r}$, $r+1$, $\frac{r}{r}$, | in the population of a province, is equivalent to | r, R, } in compound intereſt.

hence,

hence, by remark 2d upon compound intereſt, $\frac{\text{L. } 2}{\text{L. } \frac{r+1}{r}}$ = the period of doubling.

Ex. Let $\frac{1}{r} = \frac{1}{36}$, then $\frac{.30103}{.0119} = 25.3$ years, the period of doubling.

If M, the amount of the population P, in the time t, be ſought, when the ratio of the exceſs of the births above the burials to the inhabitants is $\frac{1}{r}$; in this caſe, $P \times \overline{\frac{r+1}{r}}^{t} = M$, the amount.

PROBLEM XIX.

Of the Probability of Survivorſhip.

Caſe 1ſt, Two lives being given, to find the probability of one of them fixed upon ſurviving the other.

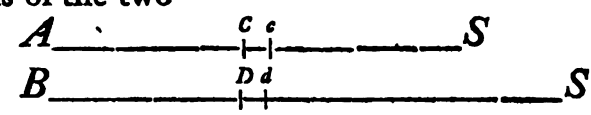
82. A — B C D — F G — S

Let AS, the complement of life, be divided into an indefinite number of equal parts, AB, BC, CD, &c. repreſenting moments of time; then will the probabilities of living from A to B, from A to C, from A to D, &c. be repreſented by the fractions $\frac{BS}{AS}$, $\frac{CS}{AS}$, $\frac{DS}{AS}$, &c. and the probability of life's failing in any interval of time AF is meaſured by the fraction $\frac{AF}{AS}$. Again, when the interval AF is once paſt, the probability of life's continuing from F to G, is $\frac{GS}{FS}$, and of its failing in that interval, is $\frac{FG}{FS}$. Hence,

The

The probability of life's continuing from A to F, and then failing from F to G, is $\frac{FS}{AS} \times \frac{FG}{FS} = \frac{FG}{AS}$; of conſequence, the probabilities of life's failing in any two or more equal intervals of time betwixt A and S, are exactly the ſame, the eſtimation being made at A conſidered as the preſent time.

Theſe things being premiſed, let the complements of the two



lives be $AS = n$, and $BS = m$; upon which take the two intervals AC, $BD = x$, as alſo the two moments Cc, $Dd = \dot{x}$; then $\frac{n-x}{n} = 1 - \frac{x}{n}$ is the probability of the firſt life's continuing to the end of the time x, or beyond it; and $\frac{\dot{x}}{m}$ is that of the ſecond's continuing from B to D, and then failing in the interval Dd; hence, $1 - \frac{x}{n} \times \frac{\dot{x}}{m} = \frac{\dot{x}}{m} - \frac{\dot{x}x}{nm}$ is the probability of theſe two events happening together at the end of the time x, or it is the fluxion of the ſum of the probabilities, whoſe fluent, viz. $\frac{x}{m} - \frac{x^2}{2nm}$ is the ſum itſelf for the time x. Let now n be put $= x$, then the probability of the firſt life's ſurviving the ſecond will be $\frac{n}{2m}$, which ſubtracted from unity, leaves the probability that the ſecond life will ſurvive the firſt.

Hence the probability of ſurvivorſhip on the part of the oldeſt life, whoſe complement is n $= \frac{n}{2m}$.

of the youngeſt, ——————— is m $= 1 - \frac{m}{2m}$.

Ex.

Ex. Let $n=46$ and $m=56$; then,

$\frac{n}{2m}=.41$ } The chance of ſurvivor. on the part of { the oldeſt.
$1-\frac{n}{2m}=.59$ } { the youngeſt.

When the lives are equal, $\frac{1}{2}$ is this probability on the part of either of them.

A, aged 60, whilſt in health, bequeaths to *B*, aged 30 years, the ſum of L. 500, in caſe he (*B*) ſhall be the ſurvivor; ſought, the preſent value of *B*'s expectation, at 4 per cent. Here,

$n=26$, $m=56$, and $P=15.983$. Hence,

$500\times\frac{P}{n}$ = L. 307.365

multiplied by $1-\frac{n}{2m}$ = .767

produces *B*'s expectation = L. 235.75

Caſe 2d, Let there be three lives, *A*, *B*, and *C*, whoſe reſpective complements are n, m, and t; it is required, to find the probability that any one of them fixed upon will ſurvive the other two.

The fluent of the expreſſion, $1-\frac{x}{n}\times\frac{x}{m}\times\frac{x}{t}$, being taken, viz. $\frac{x^2}{2mt}-\frac{x^3}{6nmt}$, and x put $=n$, we ſhall have $\frac{n^2}{3mt}$ = the probability that the oldeſt life *A* will ſurvive *B* and *C*. Again, the probability, that the ſecond life *B* will ſurvive the other two, is, $\frac{m}{2t}-\frac{n^2}{6mt}$; and theſe two, ſubtracted from unity, leave the probability of the youngeſt life *C* ſurviving *A* and *B*. Hence the probability of ſurvivorſhip on the part of

A,

A, after B and C = $\frac{n^2}{3mt}$.

B, —— A and C = $\frac{m}{2t} - \frac{n^2}{6mt}$.

C, —— A and B = $1 - \frac{m}{2t} - \frac{n^2}{6mt}$.

Ex. Let $n=40$, $m=50$, and $t=60$. Then,

.17f = A's
.32f = B's
.49f = C's
} chance of surviving { B and C. A and C. A and B.

When the lives are equal, $\frac{1}{3}$ will be the probability that any one of them fixed upon will survive the other two.

Case 3d, Let there be three lives, A, B, and C. It is required, to find the probability of survivorship on the part of any two of them after the third.

The fluent of the expression $\dot{1-\frac{x}{n}} \times \dot{1-\frac{x}{m}} \times \frac{\dot{x}}{t}$, being taken, and x put $= n$, will give the probability that the two oldest will survive the youngest, $= \frac{n}{2t} - \frac{n^2}{6mt}$. Hence the probability of survivorship on the part of

A and B after C = $\frac{n}{2t} - \frac{n^2}{6mt}$.

A and C —— B = $\frac{n}{2m} - \frac{n^2}{6mt}$.

B and C —— A = $1 - \frac{n}{2m} - \frac{n}{2t} + \frac{n^2}{3mt}$.

Ex. Let $n=40$, $m=50$, and $t=60$. Then,

.24f = A and B's
.31f = A and C's
.44f = B and C's
} chance of surviving { C. B. A.

When the lives are equal, $\frac{1}{3}$ will be the probabi-

lity that any two of them fixed upon will ſurvive the third.

Caſe 4th, It is required, to find the order of ſurvivorſhip betwixt the three lives, A, B, and C, whoſe reſpective complements are n, m, and t; or the probability that A ſhall ſurvive B, and B ſurvive C, &c.

$$\begin{aligned}
1ſt,\ \text{A, B, C,} &= \frac{n^2}{6mt}.\\
2d,\ \text{A, C, B,} &= \frac{n^2}{6mt}.\\
3d,\ \text{B, A, C,} &= \frac{n}{2t} - \frac{n^2}{3mt}.\\
4th,\ \text{B, C, A,} &= \frac{m}{2t} - \frac{n}{2t} - \frac{n^2}{6mt}.\\
5th,\ \text{C, A, B,} &= \frac{n}{2m} - \frac{n^2}{3mt}.\\
6th,\ \text{C, B, A,} &= 1 - \frac{m}{2t} - \frac{n}{2m} + \frac{n^2}{6mt}.
\end{aligned}$$

Here it is manifeſt, that theſe ſix orders, taken two and two, ſuch as the 1ſt and 2d, or the 1ſt and 3d, are reſpectively equal to the firſt equations in the two foregoing caſes, &c.

Ex. Let $n=40$, $m=50$, and $t=60$. Then,

Order 1ſt, = .08⅔	Order 4th, = .17½
2d, = .08⅔	5th, = .22½
3d, = .15⅚	6th, = .27½

When the lives are equal, $\frac{1}{6}$ will expreſs the probability of ſurvivorſhip betwixt any order of the three fixed upon.

A, aged 70 years, whilſt in health, makes a will, whereby he bequeaths L. 300 to B, aged 45, and L. 500 to C, aged 36 years, with this condition, that if either of them die before him, the whole is to go to the ſurvivor of the two: Sought, the values of the expectations of B and C, eſtimated

ted from the time that the will was writ, and reckoning intereſt at 5 per cent. Here, $n=16$, $m=41$, $t=50$, and P, correſponding to n, $=10.838$.

Was it certain that A would die before either of them,

B's expectation would be $300\times\frac{P}{n}=$L. 203.21

C's $500\times\frac{P}{n}=$L. 338.69

Which expectation of C, multiplied by the probability that he and B ſhall ſurvive A, = .6865

Produces C's expectation, ariſing from the proſpect of A's dying before either of them, = L. 216.916

But C hath a further expectation, ariſing from the proſpect that he ſhall ſurvive A, and that A ſhall ſurvive B, in which caſe he ſhall obtain 203.21

Which expectation, multiplied by the 5th order, = .1535

Produces his chance of B's legacy, = 31.193

To which add 216.916

Their ſum is the whole of C's exp. = L. 248.11

In like manner, B's expectation, ariſing from the proſpect that they will both ſurvive A, = L. 139.504

To which add his chance of C's legacy, = 40.101

Their ſum is the whole of B's expectation, = L. 179.605

PROBLEM XX.

Of Half-yearly and Quarterly Payments of an Annuity.

83. 1ſt, IN caſe of an annuity $\frac{1}{2}a$, paid every half-year, during n years, at a given rate, let P be the value of the annuity a for n years, V, the value of the half-yearly annuity for the ſame number of years, and $d = R^{\frac{1}{2}} - 1$, the intereſt of L. 1 for half a year, (Table in 46th); then

$P = \frac{a - \frac{a}{R^n}}{r}$, (47.) $V = \frac{\frac{1}{2}a - \frac{\frac{1}{2}a}{R^n}}{d}$, (49.) divide both by $a - \frac{a}{R^n}$, hence $P : V :: \frac{1}{r} : \frac{1}{2d}$, which gives $\frac{rP}{2d} = V$. If quarterly, $d = R^{\frac{1}{4}} - 1$, and $\frac{rP}{4d} = V$.

Ex. Let there be an annuity of L. 30 per annum, paid every half-year, by equal portions, for the ſpace of 20 years; ſought, the value thereof, at 4 per cent. Here $\frac{1}{2}a = 15$, $P =$ L. 407.7, and $d = .01984$. Hence $\frac{rP}{2d} =$ L. $410.96 = V$, the value ſought.

Or thus: The annuitant, in caſe of half-yearly payments, has the advantage above the annual payments of the intereſt of $\frac{1}{2}a$ for half a year, that is, of $\frac{1}{4}a$ yearly, during the continuance of the annuity; which intereſt, at 4 or 5 per cent. is equal to $\frac{1}{100}$ or $\frac{1}{80}$ part of the annuity; therefore to the value of the annuity paid yearly, add its 99th or 79th part, the ſums will be the values of the half-years payments.

2d,

2d, If an annuity is paid for life, in this caſe, half-yearly payments are $\frac{1}{4}$ of a year's purchaſe better than yearly ones.

The annuitant, by the laſt, hath the advantage of the intereſt of $\frac{1}{4}$ of the annuity during life, beſides an equal chance of receiving one half-year's value more than if he had been paid yearly; which chance is therefore, at his death, worth $\frac{1}{4}$ part of the yearly annuity. Now theſe two advantages, it is manifeſt, will always amount to $\frac{1}{4}$ of a year's purchaſe in preſent value.

Again, half-yearly payments, which begin immediately, are half a year's purchaſe better than thoſe that begin at the end of half a year; that is, by the above, they are $\frac{3}{4}$ of a year's purchaſe better than yearly payments which begin at the end of the year. But yearly payments which begin immediately are a whole year's purchaſe better than the ſame payments to begin at the end of the year: therefore the difference of value between yearly and half-yearly payments, ſuppoſing both to begin immediately, is $\frac{1}{4}$ of a year's purchaſe in favour of the former.

3d, To find how many years poſſeſſion will reimburſe an annuitant for the price paid for his annuity.

Calling n the number of years required, a purchaſer will be reimburſed for his money, when the years-purchaſe of his annuity for n years certain is equal to the value of his life; that is, when $\frac{1-\frac{1}{R^n}}{r}=N$; hence $R^n=\frac{1}{1-rN}$, and $n=\frac{\text{L.}\frac{1}{1-rN}}{\text{L.}\ R}$ which is always leſs than the expectation of life

conſidered as ſo many years certain. (Conſtruction of Table II.)

Ex. A perſon, aged 30 years, purchaſes an annuity for life: Sought, how long he muſt live before he can be reimburſed for the purchaſe-money, at 4 per cent.

Here $N = 14.684$, and $\frac{\text{L.}\frac{1}{1-rN}}{\text{L. } R} = \frac{.395}{.017} = 23.23$ years.

When an annuity is bought at a number of years purchaſe, $= \frac{1}{2r}$, in this caſe, $\frac{1}{1-rN} = 2$, and $n = \frac{\text{L. } 2}{\text{L. } R}$ the time in which money doubles itſelf at a given rate.

When the number of years purchaſe $= \frac{1}{r}$, n becomes indefinite, and the poſſeſſion muſt be a perpetuity.

PROBLEM XXI.

Of the Britiſh State Lottery.

84. The State-Lottery is a ſpecies of gambling permitted by the legiſlature, for the purpoſe of ſecuring a certain yearly revenue to government.

The Lottery anno 1791 contains 50,000 tickets; and the number and value of the prizes, begun drawing February 20. 1792, are repreſented in the following ſcheme.

SCHEME

SCHEME *of Britiſh State Lottery* 1791.

Number.	*Prizes.*		*Sum.*
2 of	30,000	=	60,000
2 of	20,000	=	40,000
3 of	10,000	=	30,000
5 of	5,000	=	25,000
10 of	2,000	=	20,000
15 of	1,000	=	15,000
30 of	500	=	15,000
50 of	100	=	5,000
100 of	50	=	5,000
14,150 of	20	=	283,000
1ſt & laſt 2 of	1,000	=	2,000
14,369 = { N° of Prizes. }		L. 500,000 = { Sum of the Prizes. }	

In this Scheme let

a = 14,369 repreſent the number of Prizes.
b = 35,631 ——— ——— Blanks.
$a+b$ = 50,000 ——— ——— Tickets.
s = L. 500,000 ——— the ſum of the Prizes.

From theſe data the following particulars are inferred.

1ſt, To find the value of each Ticket, &c.

$\frac{s}{a+b}$ = L. 10 = v, the value of each Ticket.

$\frac{s}{a}$ = L. 34.79 = V, ——— Prize.

put p = L. 16.8, the market-price of each Ticket, then $\overline{p-v} \times \overline{a+b}$ = L. 340,000, = equal Profit of government, including the expence of management, &c.

2d, To

2d, To find the chance or probability of drawing a Prize or a Blank. Here,

Seeing $a+b : a :: 1 : \frac{a}{a+b}$. Hence the following ratios.

1ſt, $\frac{a}{a+b} = \frac{1}{3.48-} = 1 : 3.48-$ } the probability of 1 Ticket drawing { a Prize.

2d, $\frac{b}{a+b} = \frac{1}{1.4+} = 1 : 1.4+$ } the probability of 1 Ticket drawing { a Blank.

3d, $\frac{a}{b} = \frac{1}{2.48-} = 1 : 2.48-$; that is, to each Prize there are 2.48 — Blanks.

Theſe ratios may be called the indices of the Lottery, which, being pointed to, or multiplied by any number of tickets, ſhow the probable number of prizes or blanks in theſe tickets.

Ex. $\frac{1}{3.48} \times 3.48 = 1 : 1$, and ſhows that 3.48 tickets have an equal chance of drawing 1 prize and 2.48 blanks; but the market-price of the tickets being L. 58.36, there is therefore a loſs of $58.36 - V = $ L. 23.57.

Again, $\frac{1}{1.4} \times 1.4 = 1 : 1$, and ſhews that 1.4 tickets have an equal chance of drawing 1 blank, and .4 of a prize: there is therefore a loſs of $23.52 - 13.92 = $ L. 9.6.

3d, To find how many Blanks or Prizes there are in n Tickets.

In n tickets there are { $\frac{n}{3.48}$ prizes, and $n - \frac{n}{3.48}$ blanks. / $\frac{n}{1.4}$ blanks, and $n - \frac{n}{1.4}$ prizes.

Ex. Let $n = 20$, there will be $\frac{20}{3.48} = 5.75$ prizes, and

and $20 - 5.75 = 14.25$ blanks. At the ſame time, the greater the number of tickets, the greater is the probability of obtaining the proportional number of prizes: Thus, 3.48 tickets have only an equal chance of drawing 1 prize; whereas $a+b$ tickets are certain of drawing a prizes: therefore the chance of drawing a proportional number of prizes varies from probability to certainty in proportion to the number of tickets.

4th, To find the probability which n tickets have of drawing a certain number of prizes, not exceeding n, let p = the prizes expected in n tickets; then $\frac{1}{p} \times \frac{n}{3.48}$ = the probability required.

Ex. Let $n = 5$, and $p = 3$; then $\frac{5}{10.44} = \frac{1}{2.09} =$ 1 : 2 = the probability of 5 tickets drawing 3 prizes.

5th, To find the probability which n tickets have of drawing any one particular prize in the ſcheme.

That 1 ticket will draw a prize

of L. 20, better than L. 20, of L. 1000 and upwards, } the probability = $\begin{cases} \frac{14.150}{a+b} = \frac{1}{3.53}. \\ \frac{219}{a+b} = \frac{1}{228.3}. \\ \frac{39}{a+b} = \frac{1}{1282}. \end{cases}$

That n tickets will draw a prize

better than L. 20, of L. 20,000, } the probability = $\begin{cases} \frac{n}{228.3} \\ \frac{n}{25,000} \end{cases}$

6th, To find the probability which any 2 tickets fixed upon have of drawing 2 capital prizes, for inſtance, of L. 10,000 each. Here, $\frac{3}{50,000} = \frac{1}{16666.\beta}$ = probability that 1 ticket fixed upon, will draw a prize of L. 10,000. But in the preſent caſe of 2 tickets drawing equal prizes, as theſe events are independent of one another, therefore $\frac{1}{16,666.\beta} \times \frac{1}{16,666.\beta} = 1 : 277,777,777.\beta =$ the probability required. Hence the vaſt diſadvantage attending ſhares of tickets drawing capital prizes. Thus the probability that 2 half tickets will draw the half of two L. 10,000 prizes, is as 1 : 277, &c. as above.

7th, To find the number of prizes and blanks drawn daily, during the continuance of the lottery, being 38 lawful days.

Each lawful day the N° of	Prizes	= 378, with	5 interſperſed.
	Blanks	= 937, ——	25
	Tickets	= 1315, ——	30
N° of L. 20 Prizes		= 372, ——	14

8th, Inſurance in the Lottery is not only expreſsly prohibited by acts of parliament, on account of the bad effects it may have upon the morals and fortunes of adventurers; but it is alſo, upon a fair calculation, ſo high, that it is not to be ſuppoſed any perſon will riſk incurring the legal penalty by ſuch a practice. Thus,

1ſt, Should an adventurer inſure, upon his ticket, the ſum *P*, not exceeding the leaſt prize, upon condition that the office ſhall make good that ſum,

ſum, in caſe of a blank, while he reſerves to himſelf the chance of drawing a prize; ſought, the premium of inſurance.

It is evident, that here only the blanks need to be inſured; therefore $P \times b$ will be the inſurance upon the whole, $a+b$ number of tickets, and $\frac{P \times b}{a+b}$ $= P \times \frac{1}{1.4}$ = the inſurance upon each ticket.

Ex. Let P = L. 16.8, then $\frac{16.8}{1.4}$ = L. 12 premium of inſurance.

Let P = L. 20, the leaſt prize, whilſt the holder retains the chance of a higher one, then, $\frac{20}{1.4}$ = L. 14.285, the premium of inſurance.

Hence any ſum P, not exceeding the leaſt prize, multiplied by the chance of drawing a blank, in any lottery, will give the premium of inſurance upon that ſum.

2d, Should an adventurer inſure upon his ticket the ſum of L. 50, whilſt he retains the chance of a higher prize; ſought, the premium of inſurance. Here $50b$ + the number of L. 20 prizes multiplied by 30 = the inſurance upon the whole number of tickets; therefore,

$\frac{50b+14150 \times 30}{a+b}$ = L. 44.12, the premium ſought.

REMARKS.

1*ſt*, In ſuch a Lottery, the true value of each ticket, excluſive of the expence of management, is L. 10; and it is owing to the confidence which the credulity of the public hath in the powers of fortune, that the market-price riſes to 16 guineas or upwards.

2*d*, The ſcheme of this Lottery is conſtructed with great art: There is a valuable bait of L. 100,000 in four prizes, hung out to catch the attention of the public; and the number of L. 20 prizes being ſo great, there are not 25 blanks to 1 prize; whereas the chance of obtaining a L. 20 prize being ſcarce diſtinguiſhable from that of obtaining a prize at all, the number of the former ought in all juſtice to be diminiſhed, and the 2 prizes of L. 30,000 each rejected, ſo as to increaſe the number of the intermediate prizes.

PROBLEM XXII.

Of a Tontine.

85. A Tontine is a kind of lottery on lives, where the capital, divided into a number of ſhares, is formed by the voluntary ſubſcriptions of thoſe who chuſe to riſk their money, in ſuch a ſcheme, upon the continuance of their lives. There are ſeveral particulars reſpecting a tontine which need to be determined.

1ſt,

1ſt, In a tontine, where 100 perſons, for inſtance, whoſe mean age is 20 years, ſubſcribe each of them L. 100, forming a capital of L. 10,000, which is ſunk to the heirs of the ſubſcribers, it is required to find the number of ſurvivors at the end of 10, 20, 30, &c. years, and what premium will then be due to each ſurvivor, reckoning intereſt at 5 per cent. Here, $n=66$, $m=100$, and $t=10$; hence, (ſupplement to problem XVIII.) $\frac{m+1\times t}{n}=\frac{1010}{66}=15.3$, who fail in 10 years; but the intereſt of the capital being L. 500, $\frac{500}{84.7}$ = L. 5.9, the intereſt due to each ſurvivor at the end of that period; hence,

living 84.7, per cent 5.90, at the end of 10 years.
——— 69.4, ——— 7.20, ——— 20.
——— 54.1, ——— 9.24, ——— 30, &c.

2d, In a tontine, where the ſhares are in proportion to the value of life, with a view to make a dividend of the capital when the ſurvivors can receive each of them L. 1000; in this caſe, if the ſhare of a ſubſcriber aged 10 or 12 years be L. 100, that of a child one year old will be L. 79, reckoning the value of life at 4 per cent; thus 16.88 : 13.36 : : 100 : 79. In like manner,

from 1 to 5 the ſhares are from 79 to 95
—— 5 to 10 —— 95 to 100
—— 10 to 18 —— 100 to 95
—— 18 to 27 —— 95 to 90
—— 27 to 39 —— 90 to 79

In ſuch a tontine, let there be 100 ſubſcribers whoſe mean age is 20 years, and the ſhares at a medium L. 90 each, forming a capital of L. 9,000; it is required, to find the time in which they

they will be ſo far reduced, that each ſurvivor can receive L. 1000. Here, $m=100$, $n=66$, and $p=91$. Hence,

$\frac{np}{m+1}=\frac{6006}{101}=59.46=t$, the time ſought.

3d, In a tontine to continue a limited time, it is required, to find the capital of each ſurvivor having a ſingle ſhare, at the expiration of that period.

Ex. In the equitable and univerſal tontine, Briſtol, commencing on the 16th day of January 1792, and continuing 7 years, wherein each ſhare is 6 s. 6 d. per quarter, with 5 d. per quarter for agency, it is required to find,

1ſt, Out of 100 ſubſcribers, whoſe mean age is 25 years, the number of lives failing in 7 years, Here $m=100$, $n=61$, and $t=7$; hence,

$\frac{\overline{m+1}\times t}{n}=\frac{707}{61}=11.6$, the number required.

2d, The amount of each ſhare at the end of 7 years, reckoning intereſt at 4 per cent. Here, $a=.325$, $R^t-1=.3159$, and $r=.00985$; hence,

$a\times\frac{R^t-1}{r}=$L. 10.425, the amount of each ſhare.

But it is evident, that, as the lives fail from year to year, the 11.6 dying out of each 100 ſubſcribers will only have contributed the half of their quota, viz. $11.6\times 5.2125=$L. 60.465, to be divided among 88.4 ſurvivors; hence,

$\frac{60.465}{88.4}=.684$, to which add L. 10.425, their ſum, viz. L. 11.11, is the capital of each ſurviyor, having a ſingle ſhare, at the end of $7\frac{1}{2}$ years.

Theſe quarterly payments, viz. 6 s. 11 d. laid out

out at 4 per cent. ſimple intereſt, where there is no riſk, would, at the end of $7\frac{1}{2}$ years, amount to L. 11.42.

SUPPLEMENT TO CHAP. VIII.

Of Inſurance from Fire.

86. IN the city of Edinburgh and its environs there are about 50,000 inhabitants; and of conſequence, by allowing 5 perſons to each family, there will be 10,000 families in that city and its neighbourhood: Now ſuppoſing that L. 100 was inſured upon each of theſe at 2 s. 6 d. per annum, this, for each 1000 families, would produce an annual revenue to the inſurance-office of L. 125.

Again, admit that 1 family out of 1000, or 10 out of the whole city, were expoſed to the accident of ſuffering loſs by fire yearly, which is a high proportion, and, we truſt, ſeldom verified by experience; this would occaſion an annual loſs to the office of L. 100 upon each 1000 families; of conſequence there would be a ſaving of L. 25 yearly, beſides intereſt, to defray the expence of management, upon each 1000 polices at 2 s. 6 d. a-piece. Were the riſks in large and ſmall properties the ſame, being in proportion to their values, their polices would be equal; but from the number of hands, fires, and lights, &c. employed, and from the connection of the parts of a building, the riſk attending L. 1000 capital is greater than 10 times that attending L. 100 value. Therefore, to find this riſk when the policy is, for inſtance, 4 s. = L. .2 per cent.; here, as

$\frac{.125 \times 1000}{.2} = 625$, the riſk is as 1 to 625.

And when the policy is 4 s. = L. .2 per cent. upon 625 capitals of L. 500 each;

The annual revenue to the office $= 5 \times .2 \times 625 =$ L. 625
The annual loſs would be = 500

And the yearly advantage = L. 125

In like manner, when

The policy is 2 s. 6d = .125		1000
3 s. = .15		833.3
4 s. = .2	the riſk is as 1 to	625
5 s. = .25		500
6 s. = .3		416.6

The number of polices = 3375

Policy.	*Capital.*	*M. Revenue.*	*Loſs.*	*Gain.*
2s. 6 d.	L. 100,	125	100	L. 25
3 s.	from L. 100 to L. 500,	312.5	250	61.5
4 s.	from L. 500 to L. 1000,	875	700	175
5 s.	from L. 1000 to L. 2000,	1875	1500	375
6 s.	L. 2000 and upwards.	2500	2000	500
		5687.5		1136.5

Add the intereſt upon the mean revenue at 4 per cent. = 237.5

Hence 3375 polices produce of yearly gain = L 1374

Of

CHAP. IX.

Of the Widows Scheme in the Church of Scotland.

87. THE fund eſtabliſhed by law, for a proviſion to the widows and children of the miniſters of the eſtabliſhed church, and of the profeſſors in the univerſities, of Scotland, is a noble monument of the knowledge, moderation, and perſeverance of the late Reverend Dr Alexander Webſter, one of the miniſters of Edinburgh, the principal contriver and promoter of this ſcheme; at the ſame time it is a ſtriking evidence of the experience which the miniſters of the church then had, of the ſtraitened circumſtances of many widows and orphans of miniſters, who, from a competency, were reduced, at once, to a ſtate of indigence; ſeeing that, out of 962 miniſters and profeſſors, only 135 declined contributing to this fund; a fund, by which, when it is complete, above 370 widows will be ſupported, and a ſum not leſs than L. 1360 diſtributed yearly among the orphans of contributors.

Previous to the eſtabliſhment of the ſcheme, ſeveral attempts had been made, by the different ſynods in Scotland, to provide for the widows and orphans of miniſters; all which proving ineffectual, from their limited nature, and for want of a common rule and proper authority to enforce it, Dr Webſter and others, about the year 1742, prepared the plan of a ſcheme, which ſhould comprehend the whole miniſters of the church, and be enforced by the authority of law. For this purpoſe they had propoſed a ſet of queries to the different preſbyteries; and from the liſts tranſmitted by them, anno 1742, in anſwer to theſe queries, founded upon an inquiry for 20 years back, the following facts were diſcovered:

1ſt, That the number of benefices in the church, independent of the offices in the four univerſities, was 970.

2d, That, taking one year with another, 27 miniſters died yearly. That 18 of them left widows, whoſe mean age was ſuppoſed to be 52 years; 5 of them children without a widow; and 4 of them neither widows nor children: Hence the expectation of a widow's life, by Dr Halley's table, viz. 16.66, multiplied by 18, produces 300, the greateſt number of miniſters widows ſuppoſed to be alive at one time.

3d, That 3 families of children, under 16 years of age, were left annually by widows of miniſters who died or married.

Theſe facts being diſcovered, the ſalaries of the general collector and clerk were eſtimated at L. 200

L. 200 yearly, which, together with a ſum not exceeding L. 39 allotted to defray the incidental expences of the truſtees, made the annual expence of management, in all L. 239; which afterwards, upon admiſſion of the profeſſors in the four univerſities to the privilege of the ſcheme, was increaſed to L. 250. And in order to depreſs the annual rates as much as poſſible, ſo as to leave room for taxes and deductions, a high rate of intereſt was aſſumed in the calculation of the rates, namely $4\frac{1}{2}$ per cent. Upon theſe principles and data, a ſcheme was prepared and laid before the General Aſſembly 1743, which approved thereof; and the parliament, upon the humble application of the Aſſembly, were pleaſed to eſtabliſh the ſame by law, and enacted that the ſcheme ſhould commence on the 25th day of March 1744, making the firſt year thereof, which ended on the 22d day of November thereafter, to conſiſt of 8 months wanting 3 days. From the liſts tranſmitted by preſbyteries at Candlemas 1745, it appeared that the number of benefices and offices in the church and univerſities, namely 1011; and conſequently the annual produce for the ſupport of the fund, was conſiderably leſs, and that the number of widows to be provided for, viz. 364, was conſiderably greater than had been ſuppoſed in the calculation of the rates previous to the aforeſaid act of parliament; and upon inſtituting a new calculation, founded upon Dr Halley's Table of Obſervations, it further appeared, that by reaſon of theſe diſadvantages, the free ſtock would become ſtationary againſt the 1771, and that L. 10,000 would then be wanting to raiſe the neceſſary capital of L. 80,000. The truſtees, for preventing this and other diſagreeable conſequen-

ces, did, purſuant to the order of the General Aſſembly 1748, petition the parliament to order certain ſums, out of the firſt and readieſt of the annual produce, to be applied from time to time towards raiſing the capital; and to authoriſe deductions, in certain caſes, from the annuities of widows and the proviſions of children. The parliament were pleaſed to enact accordingly.

During the ſpace of 30 years, from the year 1748, the affairs of the ſcheme continued, as they ſtill continue, to proſper. The truſtees, however, about the year 1773, with the advice of Dr Webſter, propoſed a number of alterations and improvements reſpecting the ſcheme to the General Aſſembly. In conſequence of which, after taking the ſenſe of the church upon theſe alterations, the Aſſembly 1778 did again petition parliament to confirm the ſame by law; the parliament were pleaſed to comply with their requeſt, by a new ſtatute, 19 Geo. III. cap. 20.

The ſeveral particulars reſpecting the perſons who are liable in payment to, or receive benefit from the fund, together with the terms of payment, the riſe of the capital, till it amounts to L. 100,000, and the duty of the truſtees, who are appointed to ſuperintend the whole, are all accurately expreſſed in the late act of parliament, which is in force from the 29th day of September 1778, a copy of which is ſuppoſed to be in the poſſeſſion of every contributor. In the preſent ſtate of the fund, in the year 1790, it is propoſed to inveſtigate the 12 following articles:

ART. 1. To find the connection betwixt the ſeveral claſſes of annual rates and their correſponding annuities in the widows ſcheme.

88.

88. According to lists transmitted by presbyteries previous to the commencement of the scheme, the benefices in the church appeared to be, as above, 970; ministers were supposed to be admitted to their benefices at the mean age of 27 years, which, by Dr Halley's table, makes their expectation of life 30 years. The widows of ministers were supposed to live, at a medium, about 20 years; and the expence of management was estimated at L. 239, which being divided by 970, quots .2464 l. for each contributor's annual share of expence. Now supposing an annuity of L. 5 was granted to each widow, or 10 years purchase of her annuity to a minister's children where there is no widow, it is required to find the annual rate which each contributor ought to pay, reckoning interest at $4\frac{1}{2}$ per cent. Here,

The present value of L. 1 annuity for 20 years,	=	13.90793
Multiplied by the annuity,	=	5
Produces,		65.03965, which
being divided by 61.00707, the amount of L. 1 annuity for 30 years quots,		1.0661
To which add the yearly expence of management,		.2464
Their sum is,		L. 1.3125 =

L. 1 : 6 : 3, the annual rate of each contributor, including his expence of management, and supposing the annuity to be L. 5. Hence, by doubling, &c. the annual rate and annuity, when

	L.	s.	d.		L.	L.
the rate is	2	12	6,	the annuity =	10, and 10 years purchase =	100,
	3	18	9,		15,	150,
	5	5	0,		20,	200,
	6	11	3,		25,	250.

From

From the conſtruction of theſe four claſſes of annual rates, it appears that, beſides bearing the burden of its correſponding annuity and yearly expence of management,

the firſt claſs is taxed with 5 s.
the ſecond with 10 s.
the third, with 15 s.
and the fourth, with 20 s. of annual expence nearly.

Theſe four claſſes of annual rates are admirable for the ſimplicity of their conſtruction; and being built upon the principles of the expectation of life and a high rate of intereſt, are as moderate as poſſible, and leave full room for taxes which are juſt, and deductions which are not oppreſſive, in order to their anſwering the purpoſes intended by the act of parliament. Thus,

1*ſt*, The rates themſelves are taxed as above, and the higher the rate the greater is the tax.

2*d*, Contributors at their marriage, and married intrants at their admiſſion, are taxed with the payment of one year's rate of the claſs to which they are ſubjected.

3*d*, The half-years rates payable out of anns and vacant ſtipends may alſo be conſidered as a tax, ſeeing the decrement of life in the incumbent is then ſuſpended; beſides the advantage which the fund receives from $\frac{4}{30}$ of the contributors who die yearly leaving neither a widow nor children, and from the expectation of a miniſter's life being $2\frac{1}{2}$ years greater than is ſuppoſed in the calculation of the rates. Again,

4*th*,

4th, There is a deduction of one years purchaſe out of the proviſion of a widow, ſeeing her annuity begins to run from a full year after the laſt payment of her huſband.

5th, There is a deduction of two years purchaſe out of the proviſion of children, which ~~is ſo much~~ leſs than that of a widow.

6th, There is alſo a deduction out of the proviſions of the widows and children of thoſe contributors who have not paid annual rates to the amount of three years of their widows annuity, without reckoning intereſt thereon, till the balance is recovered; beſides the advantage which the fund derives from widows, not having children under 16 years of age, who happen to marry not-contributors, and of conſequence are ſtruck off from the benefit of the fund: Whereas the only advantage which the ſcheme affords above the calculation, is the reverſion of 10 years annuity to the children under 16 years of age, of thoſe widows who happen to die before they have completed the 11th year of their widowhood.—Theſe rates, taxes, and deductions, together with the proviſion in the act of parliament to ſecure the riſe of the capital to L. 100,000, by the application of L. 200 yearly for this purpoſe, will, we truſt, give a ſtability to the ſcheme, and make it anſwer all the purpoſes for which it was intended.

In the conſtruction of theſe rates, ſuppoſing the number of benefices and offices in the church and univerſities to be 1013, including the third charge in Perth and the ſecond in Linlithgow, and fixing the annual expence of management, agreeable

greeable to the two firſt acts of parliament, at L. 250, we ſhall have $\frac{250}{1013}$ = L. .2467 for each contributor's ſhare of expence, which, for an annuity of L. 5, will alſo produce a rate, as above, of L. 1 : 6 ; 3, nearly exact.

Had theſe rates been calculated upon the ſtrict principles of reverſionary annuities, the calculation would have been more laborious, and the rates and marriage-taxes much higher than they are at preſent in the widows ſcheme. Thus:

Suppoſe a contributor, aged 30, marries a wife of 25 years of age, whoſe annuity is L. 20; ſought, the annual rate which he ought to pay, reckoning intereſt at 4 per cent. Here,

$\frac{3.89}{14.684} \times 20 + .2464 \times 4 = \text{L. } 6.23$, the annual rate required.

Again, If the ſame contributor becomes a widower, and, at the age of 40, marries a ſecond wife aged 30 years, his marriage-tax would be L. 8.68. See Prob. VIII. Of a Widows Scheme.

Theſe annual rates are due to the fund, either for a whole or an half year, at the ſame terms that an incumbent hath a right to his ſtipend or ſalary for the ſame periods, and are payable to the collector at the firſt term of Candlemas thereafter; excepting in the caſe of the half-years rates payable out of anns and vacant ſtipends, the former of which is always half of the contributor's rate, and the latter is fixed at L. 3, 2 s. for each half-year of vacancy, theſe, though due at the ſame terms with the former, are not payable to the collector till the firſt term of Candlemas which ſhall happen a full year after the anns and vacant ſtipends ſhall be due and payable. The medium of

of annual payments by contributors being within a trifle of the third claſs of L. 5, 5 s. this third claſs is eſteemed, in all calculations reſpecting the ſcheme, as the medium of the annual rates of all the contributors, and L. 20 the medium of all the widows annuities.

ART. 2. To find the number of benefices in the church, and offices in the univerſities, which are all ſubject to the payment of one or other of the annual rates.

89. By the liſts tranſmitted to the truſtees, by preſbyteries and univerſities, it appeared that, at Martinmas 1744, there were,

Of Benefices in the Church, 942
And of Offices in the Univerſities, 69 } 1011 in all.

Of the ſaid Benefices and Offices, { full 962 / vacant 49 } 1011 in all.

From the beſt information which I have received, it appears that, at Martinmas 1789, there were,

Of Benefices in the Church, 943
And of Offices in the Univerſities, 68 } 1011 in all, as above.

It being left optional, by the firſt act of parliament, to the miniſters of the church then living, to accede to, or decline the privilege of, the ſcheme; there were, at Martinmas 1744, 135 decliners, that is, perſons who declined accepting the benefit of the ſcheme, conſequently there remained only 827 original contributors; and as not only their ſucceſſors, but the ſucceſſors alſo of the 135 not-contributors are obliged to become ſubject to the ſcheme, the greateſt number of contributors alive at one time will be 962, making

in all, with the addition of 49 vacancies, 1011 benefices in the church and univerſities liable to the payment of one or other of the annual rates. Hence, as the above number of benefices alſo includes the anns, the greateſt ſum of annual rates paid in any one year, excluſive of marriage-taxes, will be $1011 \times 5.25 = L.\ 5307, 15s.$

As ſeveral of the not-contributors, at the commencement of the ſcheme, were below 30 years of age, the number of contributors will not be complete till about the 60th year of the fund; and there being 7 of thoſe alive at Martinmas 1789, the number of contributors may be placed in this form in the ſucceeding years of the fund, at Martinmas yearly.

Year.	*Fund.*	*Contribut.*
1790	47	955
1791	48	955
1792	49	957
1793	50	958
1794	51	959
1796	53	960
1799	56	961
1803	60	962

Although there be ſeveral miniſters of the church, who alſo hold offices in the univerſities; yet this diminution of the number of contributors, ſuch not being liable to double rates, is more than counterbalanced by the number of thoſe who have reſigned their benefices or offices, and are ſtill ſubject to the fund.

From the liſts tranſmitted by preſbyteries anno 1742, it appeared, as mentioned above, that out of 897, the greateſt number of miniſters in life at one

one time, 27 died yearly, &c. According to which proportion, out of 962 miniſters and profeſſors, 28.956 would die yearly: of whom 19.3 would leave widows, 5.36 children without a widow, and 4.29 neither widows nor children.

From the truſtees report for the 46th year of the fund, ending on the 22d day of November 1789, it appears that there have died of miniſters and profeſſors, ſince the commencement of the ſcheme, 1345

Which being divided by the number of years, 45.663 quots 29.455,

the number who have died yearly.

Hence the number alive at one time, = 962

Divided by the number who die yearly, = 29.455 quots 32.66,

the expectation of a miniſter's life.

From the ſame report it appears, that 19.3 left widows, 5.8 children without a widow, and 4.4 neither widows nor children, yearly. But ſeeing, by Dr Halley's table, out of 962 perſons of 27 years of age and upwards, 31.79 would die yearly; Dr Webſter, in his calculations, ſuppoſes that 30 miniſters and profeſſors would die yearly; out of whom 20 would leave widows, 6 of them children without a widow, and 4 of them neither widows nor children. He ſuppoſes further, that the mean age of widows, at the death of their huſbands, was 52, and that the expectation of their life was 20 years.

ART. 3. To find the annual number of thoſe

contributors who are liable to the payment of the marriage-tax.

90. As 30 are ſuppoſed to die annually out of the whole miniſters and profeſſors, of whom 26 leave widows or children; and in regard many wives predeceaſe their huſbands, and ſeveral of them leave no children; we cannot ſuppoſe fewer than 28 to marry annually, who, in conſequence of their marriage, except to annuitants, are liable in a ſum equal to one year's rate of the claſs to which they belong; the juſtneſs of which inference depends upon the ſuppoſition, that all intrants are bachelors, and that there are no ſecond marriages. Therefore, independent of conjecture, it is found in fact, that there are about 26 incumbents who marry yearly in the church and univerſities; and that there are about 6 married perſons admitted yearly, for the firſt time, to benefices and offices, making in all 32. Beſides, of thoſe who are admitted into the church and univerſities, there are 5 or 6 who are 40 years of age or upwards. Now, ſuppoſing that the half of theſe are either married, or afterwards do marry, and, in conſequence of their marriage, are, by the late act, liable in a ſum equal to $2\frac{1}{2}$ years rates of the claſs to which they belong, this will increaſe the annual number of marriage-taxes to about 35. Hence the ſum of marriage-taxes paid annually by contributors will be $35 \times 5.25 = $ L. 183, 15 s. Moreover, 32, the number of yearly marriages, being multiplied by 19.43, the expectation of marriage between two perſons of 30 and 25 years of age reſpectively, will produce 622, the greateſt number of married perſons in the church and univerſities at one time; which number being equal

to

to that of all the widows and widowers in life at one time, and the proportion of theſe to one another being as 19.4 to 12.6, we ſhall have 32 : 19.4 :: 622 : 377, the greateſt number of widows in life at one time. See the following Article.

A marriage-tax is payable to the collector, at the firſt term of Candlemas which ſhall be one full year after marriage; or, in the caſe of a married intrant, after his admiſſion to a benefice or office.

ART. 4th, To find the greateſt number of annuitants which can come upon the fund. This may be done, 1ſt, by the annual reports of preſbyteries, which, among other articles, contain liſts of all the widows alive at Martinmas each year, the medium of which liſts for a few years, could we depend upon their accuracy, would ſhew the maximum of annuitants. Thus, during a ſpace of 20 years from Martinmas, 1749 their mean number was below 380; and the number of widows, annuitants, and others, alive at Martinmas 1789, was 376, including 4, which are added upon account of the great number of families of children of the firſt order, being 8 more than uſual, which came upon the fund at Whitſunday 1781.

2d, By their expectation of life when they commence widows. Let the reſpective ages of huſband and wife be 30 and 25 years; then, to the age of the wife, viz. 25
add the expectation of marriage, by Dr Halley's table = 19.5
their ſum, increaſed by .5 on account of ſecond marriages, gives 45, the mean age of widowhood; therefore 19.56, the expectation of a perſon's life aged

aged 45, multiplied by 19.3, the number left yearly, will produce 377.5, the maximum of annuitants; whereas 16.66, the expectation of a perſon's life aged 52, multiplied by 20, the number of widows ſuppoſed to be left yearly, produces only 334, which is the number Dr Webſter brings out, in the third table of his calculations. So ſenſible was the Doctor of the defect of this maximum, that, from the year 1781, he ſuperadds, to the number of annuitants by calculation, no fewer than 66, making in all 400 in life at one time.

3d, By the numbers already upon the fund. Thus at Whitſunday 1779, the number of annuitants drawing full was 289.5, including heirs, beſides 9 drawing half annuities; at which term there were 181 benefices which had caſt no widows upon the fund, but which, in the ſpace of 11 years, produced 34 additional ones, including 4 which are added for a reaſon mentioned above; at Whitſunday 1790, there were 123 benefices which had produced no widows to the fund. Hence,

181 : 34 :: 123 : 23, the number of additional widows at Whitſunday 1801, making in all 342.5, including heirs, drawing full, and 10 drawing half annuities, when the number of contributors will be complete; at which period there will ſtill be 52 benefices unproductive of widows; hence,

962—52 : 342.5 :: 52 : 19.5 the number of widows ſtill to come upon the fund after Whitſunday 1801. Hence the maximum of annuitants will be 362, including heirs, drawing full, and 10 drawing half annuities; that, is there will be 362—5 = 357 drawing full and 10 drawing half annuities,

annuities, together with 10 not entered upon the fund; in all 377, excluſive of heirs.

It is ſomething remarkable, that although, in aſſuming the expectation of a widow's life from 52 years and upwards, there is a defect of 2.9 years, yet Dr Webſter's calculation, Table III, agrees nearly exact with the fact, till about the year 1781. To account for this I obſerve,

1ſt, That the Doctor, in ſuppoſing 20 widows to be left yearly, and that theſe are all alive at the following Martinmas, aſſumes $.7+.3=1$ more than the fact; which circumſtance would increaſe the number of annuitants by 16, when that number was complete.—2d, That, from the nature of the calculation, he ſuppoſes the number of contributors to be complete in the year 1781; whereas there were 15 decliners ſtill alive, and, by proportion, there were 107 original contributors and others ſtill in life, who would produce 37 widows, ſuppoſing the maximum to be 334.

3d, That the Doctor's calculation comprehends all the widows alive whether married the ſecond time or not; which circumſtance may increaſe the number about 10 more than the fact, when they arrive at a maximum; now the ſum of theſe, viz. $16+37-10=43$, added to 334, will give 377, the true maximum of annuitants upon the fund; or, ſubtracted from the true maximum, will leave 334, the number of widows by the Doctor's calculation, Ann. 1781, ſuppoſing there had been 962 contributors from the beginning. The induction of theſe particulars is a further proof that the maximum found in this article is near the truth.

The number of annuitants cannot be complete, not only till after the death of all the decliners, which may happen about the 60th year of the fund, but

but alſo while any of their ſucceſſors in office are alive, which may be till about the 120th year of the ſcheme; during the laſt 60 years of which period, there are only about 20 additional widows to come upon the fund. Hence the number of annuitants, including heirs, drawing full annuities, at the term of Whitſunday yearly, beſides 10 drawing half annuities, may be repreſe nted by the following table.

Years.	Fund.	Annu.	Years.	Fund.	Annu.
1790	47	319	1806	63	345
1	48	22	7	64	46
2	49	24	9	66	47
3	50	26	1811	68	48
4	51	28	13	70	49
5	52	330	15	72	350
6	53	32	17	74	51
7	54	34	19	76	52
8	55	36	1821	78	53
9	56	38	24	81	54
1800	57	40	27	84	55
1	58	42	1830	87	56
2	59	43	33	90	57
4	61	344	1863	120	362

A widow's annuity is computed to run from the 26th day of May, or the 22d day of November, which ſhall be one full half year after the huſband's death; and the firſt year's or half year's annuity is payable on the 26th day of May, which ſhall be a full year, or a full half year, reſpectively, after the time from which the annuity is computed to run; and it continues payable on the 26th day of May yearly, during the widow's life, and her remaining unmarried; excepting that, by the late act of parliament, annuitants, having no children under 16 years of age, and marrying contributors,

contributors, are entitled to one half of their annuities, until the capital is completed; after which they ſhall enter upon their full annuities; and as there is nothing due to her heirs for the half year in which an annuitant dies, the heirs only of thoſe who die from Martinmas to Whitſunday are entitled to half years annuities at the following term of Whitſunday.

As a widow is ſuppoſed to live at a medium 20 years, did ſhe come upon the fund immediately at her huſband's death, her annuity, reckoning intereſt at $4\frac{1}{2}$ per cent. would be worth 13 years purchaſe; but as ſhe hath no claim upon the fund till three quarters of a year at a medium after the above term, and as ſhe loſes one quarter of a year at her own death, her annuity is only worth 12 years purchaſe.

ART 5. Reſpecting families of children of the firſt order.

By theſe are underſtood thoſe families of children whoſe fathers at their reſpective deaths left no widows upon the fund.

89. In calculations reſpecting the ſcheme, it is ſuppoſed that 6 ſuch families of children are left yearly, who, by act of parliament, are intitled to 10 years purchaſe of their fathers widows annuity, that is in all, to the ſum of L. 1200. And the proviſion of each family is divided equally among them where there are more children than one; which proviſion of children falling due in any one year, computed from the 22d day of November to the 22d day of November in the year following, is payable by the collector, on the 13th day of Auguſt thereafter.

ART. 6. To find the number of families of children of the ſecond order which come yearly upon the fund, that is, children, who being under 16 at the death or marriage of their reſpective mothers, are intitled to the reverſion of what they have not drawn of 10 years annuity.

90. According to liſts tranſmitted by preſbyteries, anno 1742, it appeared, as mentioned above, that 3 ſuch families were left annually by widows of miniſters who died or married, having children under 16 years of age. Now although the ſame number die, yet it is probable that fewer marry ſince, than before the commencement of the ſcheme, on account of the loſs of their annuities; we will ſuppoſe that there are 2.5 families of children left annually by widows who died, or married, having a child or children under 16 years of age. But it is evident that no widow who ſurvives her huſband 17 years, can leave a family in theſe circumſtances, and by the act of parliament, it is only the children under 16 years of age, of ſuch widows as have not drawn 10 years annuity, or have not ſurvived the 11th year of their widowhood, who are intitled to the reverſion; therefore,

17 : 2.5 : : 11 : 1.6 the number of families of children of the ſecond order, who are yearly intitled to the reverſion of their reſpective mothers annuity; which reverſion is worth from 0 to 10, that is 5 years purchaſe; Therefore, $1.6 \times 100 =$ L. 160, is the yearly ſum due to families of children of the ſecond order.

In this article I differ from Dr Webſter, who makes the number of families of children of the ſecond order to be 1.5, becauſe, from the nature of

of the calculation, the Doctor's number does not comprehend thoſe widows who may happen to marry, having children under 16 years of age; whereas I include theſe, calling them .5 yearly, ſeeing they are excluded from the number of widows actually upon the fund. At the ſame time, I avoid the ſuppoſition, that any woman, aged 69 years, can leave a child or children under 16 years of age.

ART. 7. To find the yearly value of the deductions from the proviſions of the widows and children of ſuch contributors as happen to die before they have paid a ſum equal to three years annuity, correſponding to the annual rate to which they were ſubjected, without computing intereſt thereon.

91. Such a deduction will take place, unleſs a contributor at his death hath paid a ſum, including his marriage-tax, amounting to $11\frac{1}{7}$ years rates, or L. 60. Now as contributors, who are admitted into the church or univerſities at the mean age of 27, cannot be ſuppoſed to marry ſooner, at a medium, than the third year of their incumbency, or in the 30th year of their age, we may conclude that the propoſed deductions will take place only with reſpect to the families of thoſe contributors who ſhall happen to die in the third, and in any ſubſequent year before the 12th of their incumbency, or in the 30th, and in any ſubſequent year before the 39th of their age. Now, by Dr Halley's table, out of 16669, the number of perſons alive of 27 years of age and upwards, 76 die annually in the 30th to the 38th year of their age, both included; according

to which proportion, 4.386 contributors will die annually in the 30th to the 38th year of their age, both included, out of 962 miniſters and profeſſors. But $\frac{1.5}{29.5}$ parts fewer of contributors die yearly, than according to the proportion of the above table, and $\frac{4.4}{29.5}$ parts of thoſe who die, leave neither widows nor children; which $\frac{5.9}{29.5}$ parts of 4.386, ſubtracted from the ſame, leave 3.509 the number of contributors dying yearly in the 30th to the 38th year of their age, leaving widows or children; each of whom have paid, including their marriage-tax, from 3 to 11, that is 7 years rates, making in all $24.56 = L.128.94$; but they ought to have paid $3.509 \times 60 = L.210.54$; therefore, ſubtracting one from the other, there remain L. 81, 12s. the yearly deductions.

However inconſiderable theſe deductions may appear, yet of ſuch conſequence were they in the early periods of the fund, that an act of parliament was obtained, anno 1748, authoriſing the ſame, in order to prevent a deficiency in the capital of L. 10,000, which was calculated to happen about the year 1771, which defect they have prevented. At the ſame time it is provided, that every widow ſhall have at leaſt one half of her annuity for ſubſiſtence until the deficiency be made good, and then enter on full payment.

ART. 8. Reſpecting the annual expences of management.

92. In the firſt draught of the ſcheme, which extended only to the miniſters of the church, theſe expences were limited to L. 239. In the two acts

acts of parliament eſtabliſhing the fund, wherein the profeſſors in the four univerſities were alſo comprehended, the annual expences were fixed at L. 250; namely L. 210 for the ſalaries of the general collector and clerk, and L. 40 for the annual expences of the truſtees. By the late act of parliament anno 1778, the ſalaries of the collector and clerk are the ſame as formerly, but the incidental expences of the truſtees are augmented to L. 70, with a proviſo that they do not in any one year exceed that ſum. Which expences of the truſtees, in imitation of Dr Webſter, are not brought as a charge upon the fund, from the ſuppoſition that the intereſt received by the collector in any one year, doth ſo far at leaſt overbalance the intereſt of the capital for that year, computed at 4 per cent.

ART. 9. Reſpecting the capital of the fund.

93. Had there been no capital, the rates being adjuſted to the annual demands upon the fund, it is evident that during the firſt 14 years of the ſcheme the annual rates would have been too low, and that afterwards, till the number of annuitants arrived at a maximum, they would have riſen gradually to L. 8.5 at a medium, which is out of all proportion to the value of an annuity of L. 20. By the act of parliament eſtabliſhing the fund, anno 1744, when the maximum of annuitants was ſuppoſed not to exceed 321, the capital was limited to L. 63,860, including the loans of L. 30 lent to contributors; the intereſt of which, together with the annual rates and taxes, was thought ſufficient to diſcharge all demands upon the fund. By the act of parliament, anno 1748, when it was found that the number

number of annuitants would exceed 364, the capital was extended to L. 80,000, including the loans of L. 30, granted to contributors at 4 per cent. And by the late act, anno 1778, when it was ſuppoſed that the number of widows might riſe to 400, the capital is allowed to be raiſed to the ſum of L. 100,000, to be lent out upon heritable ſecurity, hitherto at $4\frac{1}{2}$ per cent.; and the loans of L. 30 to contributors are prohibited, which of conſequence will all revert to the capital about the 95th year of the fund. And in order to ſecure the riſe of the capital to L. 100,000, the truſtees are directed to take care that the ſum of L. 200, together with the ſavings and ſurpluſes of intereſt, annual rates, &c. that theſe, after all demands upon the fund are diſcharged, be applied annually for that purpoſe, by lending out the ſame, with the advice and conſent of the learned judges who preſide in our courts of law, until the capital is made up. When it is completed, there will be an overplus, reckoning intereſt at $4\frac{1}{2}$ per cent. after paying all demands upon the fund, of about L. 1400 yearly; of which L. 150 muſt be reſerved as a proviſion for thoſe 18 widows who are ſtill to come upon the fund: as for the remainder, the church muſt conſult and determine in what manner it ought to be diſpoſed of, when the capital is raiſed to L. 97,000.

94. ART. 10. To trace the progreſs of the capital from L. 87751.522, in the 46th year of the fund, till it ariſe to L. 100,000. Here,

Art.

Art.

2. The number of contributors, being from 955 to 962.
3. —— of perſons liable to the marriage-tax, 35.
2. —— of vacancies, 49.
4. —— of annuitants, including heirs, drawing full, beſides 10 drawing half annuities, being from 319 to 362.
5. —— of families of children of the firſt order, being 6, = L. 1200.
6. —— of families of children of the ſecond order, being 1.6, = L. 160.
7. Deductions from the proviſions of widows and children, = L. 81.6.
8. The ſalaries of the collector and clerk, = L. 210.
1. The medium of annual rates, L. 5.25; and of annuities, L. 20; and the rate of intereſt upon the capital 4 per cent. The progreſs of the capital may be repreſented by the following table.

I.	II.	III.	IV.	V.	VI.	VII.
1789	46	Rates.	their prod.	Ann.	their exp.	87,751.522
1790	47	1039	5454.75	319	7968.4	88,747.932
1791	48	1039	5454.75	322	8028.4	89,724.2
1792	49	1040	5460.	324	8068.4	90,704.768
1793	50	1041	5465.25	326	8108.4	91,689.809
1794	51	1042	5470.5	328	8148.4	92,679.5
1795	52	1043	5475.75	330	8188.4	93,674.03
1796	53	1043	5475.75	332	8228.4	94,668.34
1797	54	1044	5481.	334	8268.4	95,667.67
1798	55	1044	5481.	336	8308.4	96,666.97
1799	56	1044	5481.	338	8348.4	97,666.25
1800	57	1045	5486.25	340	8388.4	98,670.75
1801	58	1045	5486.25	342	8428.4	99,675.43
1802	59	1045	5486.25	343	8448.4	100,700.3
1803	60	1045	5486.25	343	8448.4	101,762.16
1804	61	1046	5491.5	344	8468.4	102,855.75

In

In this Table, Column VII. ſhews the amount of the free ſtock at Whitſunday for each correſponding year of the fund in Col. II. the intereſt of which, computed at 4 per cent. together with the ſum of the rates and taxes of the following year, Col. IV. is added to ſaid ſtock, and their ſum, diminiſhed by the charge upon the fund, Col. VI. is the free ſtock of the following year. Col. III. ſhews the number of rates and taxes due at Candlemas, for each year of the fund; and Col. V. the number of annuitants, including heirs, drawing full, beſides 10 drawing half annuities at the term of Whitſunday yearly.

Ex. To ſtock at Whitſunday 1801,			= L. 99675.43
Add intereſt to Whitſunday 1802,			= 3987.02
And 1045 rates and taxes,			= 5486.25
Total,		=	L. 109148.7
Deduct annuities diminiſhed by deductions, Art. 7.	= L. 6878.4		
Childrens proviſions and ſalaries,	= 1570.0		
			8448.4
Stock at Whitſunday 1802,		=	L. 100700.3

Thus, about the 61ſt year of the fund, the capital will be made up, the number of contributors will be complete, and there will be 18 widows ſtill to come upon the fund in the courſe of 60 years.

ART. 11. Reſpecting alterations upon the ſcheme.

97. 1ſt, It is required, to find what capital would

would be neceſſary to ſupport the fund upon its preſent footing, when the annuitants are arrived at a maximum, reckoning intereſt at 4 per cent.

From the widows annuities,	= L.	7340
Childrens proviſion and ſalaries,	=	1570
Their ſum,	=	8910
Deduce rates, taxes, and deductions,	=	5573.1
Divide by .04	) 3336.9 (	L. 83422.5, the Capital.

2d, It is required, to find what diminution might be made upon the annual rates, ſuppoſing the annuities to remain the ſame as at preſent, when the capital is raiſed to L. 100,000, reckoning intereſt at 4½ per cent.

To 357 annuitants drawing full,	= L.	7140
Add 20, including heirs, drawing half annuities,	=	200
Childrens proviſion,	=	1360
And ſalaries,	=	280
Their ſum,	=	8980
Deduce intereſt and deductions,	=	4581.6
Divide by rates and taxes, = 1046	) 4398.4 (	L. 4.2, Annual Rate.

Here, $1046 \times 5.25 - 4398.4 = L.\ 1093.1$, is the annual ſurplus.

3d, It is required, to find what augmentation might be made upon the annuities, the annual rates remaining the ſame as at preſent, when the capital is raiſed to L. 100,000, and reckoning intereſt at $4\frac{1}{2}$ per cent.

From intereſt, rates, and deductions, = 10073.1
Deduce ſalaries, = 280.0
L.
Divide by annuitants and children, = 435) 9793.1 (22.51, the Annuity.

ART. 12. Containing a general view of the ſcheme.

96. 1ſt, It is peculiar to this ſcheme, that the annual rate varies, not with the age of a contributor, but only in proportion to the annuity; that every intrant who is married, or afterwards marries, is liable to a marriage-tax; that its conſtruction is built upon the principles of the expectation of a miniſter's life, which, being above 34 years, including the vacancy, with a high rate of intereſt, depreſſes the rates as much as poſſible; that the widow's annuity begins to run, from a full twelvemonth after the laſt payment of the huſband; that widows who happen to marry are deprived of the whole, or of part of their annuities; and it is owing to this ſcheme that the diſtinction of families of children of the ſecond order was ever heard of.

2*d*, As to the diſadvantages attending this ſcheme, if there be any, they are ſuch as are in common with all laudable undertakings of this kind.

kind. Thus the leading principle of ſuch a ſcheme not being juſtice and equity, but charity, mercy, and compaſſion for the indigent; the more you pay, the leſs is received, and the leſs you pay, the greater is the value of the annuity. As all intrants are obliged to contribute for the ſupport of the fund, this circumſtance, though an advantage to the ſcheme, is often a hardſhip upon the individual, who is thus obliged to provide for a widow, when poſſibly he may have no thoughts of taking a wife. As heirs in general diſtinct from widows and children have no intereſt in the fund, the contributions of $\frac{4}{30}$ parts of thoſe who die yearly are loſt to them and their heirs for ever, if what is given for the ſupport of the widow and fatherleſs can be called a loſs. The depriving a widow of her annuity in conſequence of a ſecond marriage, may be conſidered as a diſadvantage attending the ſcheme, not from a principle of juſtice or charity, ſeeing every woman who marries is ſuppoſed to be ſupported by her huſband, but in a political view, as the increaſe of ſociety is thereby ſo far diſcouraged. And the capital of the fund, like all other ſums of money laid out at intereſt, is liable to a diminution in its value, from the poſſible fall of the intereſt of money. But I inſiſt, with much more pleaſure,

3*d*, Upon the advantages attending the ſcheme; as hereby, in common with other inſtitutions of the ſame kind, ſo many widows and orphans receive the means either of ſubſiſtence or of education, which muſt prove a benefit to contributors themſelves, who, in conſequence of the ſcheme, are enabled to marry with greater advantage than

they could have done had it not taken place. The annual rates are exceeding low, on account of the advantages which the ſcheme itſelf enjoys; as, 1ſt, All entrants being ſubject to the ſcheme, they enter early, and of conſequence have a chance of continuing long contributors to the fund. 2d, Bachelors and widowers being liable to the payment of the annual rates, the heirs of a ſeventh part of thoſe who die yearly derive no advantage from the fund. 3d, Anns and vacant ſtipends, where there is no waſte in the life of the incumbent, are ſubjected to the payment of half years rates. 4th, There are a number of taxes and deductions, ſee Art. 1. which are not ſuppoſed in the calculation of the rates. 5th, Should a contributor fail in the payment of his rates, which are privileged debts, there is a viſible fund, namely, his ſtipend or ſalary, out of which the ſcheme may be reimburſed for all loſſes occaſioned by him; and ſhould this fail, the collector hath 26 chances out of 30, of recovering at 5 per cent. ſimple intereſt, all arrears due by a contributor, out of his widow's or childrens proviſion. 6th, The riſe of the capital to a certain amount, which is lent out upon heritable ſecurity, and its recovery, ſhould it ſuſtain any material loſs, are both ſecured by act of parliament. 7th, The number of contributors is fixed and determined, being limited to the number of benefices in the church and offices in the four univerſities; ſo that the truſtees are enabled to foreſee what demands are likely to be made upon the fund from year to year. 8th, The ſcheme itſelf hath undergone ſeveral alterations and improvements, and may ſtill undergo more by favour of the Britiſh legiſlature. 9th, The whole concerns of the ſcheme are under the

the direction and management of truſtees, moſt reſpectable for their knowledge, experience, and perſeverance in the affairs of the fund; namely, the Reverend Miniſters of the preſbytery, and learned Profeſſors in the univerſity of Edinburgh, together with the miniſters in the preſbytery ſeats, or who are admitted to offices in the univerſities, if they choſe to accept, and delegates from preſbyteries and univerſities, who, did they all convene, would form a general court of directors, as reſpectable for their numbers and importance as the Venerable Aſſembly itſelf.

Therefore, I cannot conclude without expreſſing my reſpect and gratitude to the Reverend and learned Truſtees, who are appointed to manage and direct the operations of the fund, and thanking them for their fidelity and unwearied application in conducting the concerns of a ſcheme, whereby the widow's heart is made to ſing for joy.

SUPPLEMENT *to* CHAPTER IX.

A

DISSERTATION

RESPECTING

The DISPOSAL of the SURPLUS in the Fund of the WIDOWS SCHEME, when the Capital is raiſed to L. 100,000; addreſſed to the Truſtees for managing ſaid Fund.

Mr PRESES,

99. THE queſtion now under conſideration, reſpecting the diſpoſal of the ſurplus in the fund of the widows ſcheme, is of a ſimilar nature with, but of a different tendency from that determined by the church, anno 1748, and of conſequence ought to have a ſimilar though oppoſite determination. In the year 1748, the Reverend Dr Alexander Webſter, the principal contriver and promoter of the widows ſcheme in the Church of Scotland, diſcovered by calculation, that there would be a deficiency in the capital of the fund, of L. 10,000 about the year 1771. What was then to be done to prevent this deficiency? Muſt the rates be raiſed? No; Muſt the annuities be diminiſhed?

niſhed? No; Touch not, would he ſay, either of theſe pillars of the ſcheme, left by ſapping and undermining the foundation, you make the whole fabric tumble about your ears. What then? Why, the Doctor did the only thing which he ought to have done; he did an act of juſtice, by obliging every contributor to pay at leaſt three years purchaſe of the annuity correſponding to the rate to which he was ſubjected, before his widow or children could be entitled to their reſpective proviſions.

In the preſent caſe, when the capital is raiſed to L. 100,000, the annual ſurplus may be about L. 1473, reckoning intereſt at $4\frac{1}{2}$ per cent.* What muſt be done to prevent the farther increaſe of the capital? Shall we diminiſh the rates in general? or ſhall we increaſe the annuities? Either, or both of theſe would be deſirable, did we not thereby perpetuate a very great hardſhip upon aged contributors, by continuing the payment of the rates till the cloſe of life; when we have it in our power to redreſs it.—But,

To find how this matter may be determined to the ſatisfaction both of contributors and annuitants, by giving each party a ſhare of the ſurplus, nearly in proportion to their numbers. There may be 435 annuitants upon the fund, when they arrive

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* When the capital is complete, an. 1802, there may be 338 annuitants drawing full, and 20, including heirs, half annuities; Hence to rates, taxes, and deductions, = L. 5573
Add the intereſt of the capital at $4\frac{1}{2}$ per cent. = 4500

Their ſum = L. 10073

To the ſum of annuities paid to widows, = L. 6960
Add chil. prov. and expence of management = 1640 } 8600

Their difference is the annual ſurplus = L. 1473

at

at a maximum, including both orders of children, which are equal in expence to 68 widows; and the number of contributors alive at one time is 962.

Now it is obvious that annuitants of all deſcriptions, whether old or young, are equally the objects of charity, and of conſequence, ought to be benefited only by the plan of augmentation; ſuppoſe by 5 per cent, or by 1 ſhilling upon the pound of their annuities; this will exhauſt L. 435 of the above ſurplus. Again,

With reſpect to contributors, in the relation they ſtand in to the ſcheme, by the payment of the rates, &c. take the following table.

Years of minr.	*Contributors.*	*Amount at 4 per cent.*	*yrs pur.*
From 0 to 12	3.6 fail who pa.	L 67.5 for an annuity worth	15.0
12 to 28	7 4	149	13.5
28 to 65	18.45.	524	10.0
0 to 33	15.0	732	14.0
33 to 55	15.0	605	10.0

By this table, out of thoſe who are alive, during the firſt 12 years of their miniſtry, 3.6 fail yearly, who, allowing 5s. for expence of management, have paid at a medium, including deductions, the ſum of L. 67.5, for a L. 20 annuity worth 15 years purchaſe. It would therefore be highly improper to grant a diminution of the rate during this ſtage of life, not only on account of the ſmall value paid for the annuity, but alſo becauſe the more the rate was diminiſhed, the greater would be the deduction out of the widows proviſion. In like manner, in the caſe of thoſe who are alive from the 12th to the end of the 28th year of their miniſtry *, when the full value of the annuity is

* To find the time in which a rate of L. 5 per annum, allowing 5 s. for expence of management, will diſcharge the augmented annuity of L. 21, worth 12 years-purchaſe, at 4 per cent. $\frac{S}{a} = \frac{252}{5} = 50.4 = N$ correſponding to 28.13 years.

diſcharged

diſcharged, it would alſo be unneceſſary to diminiſh the rate during this period, becauſe contributors have not paid the full value of the annuity. But in the caſe of thoſe who are alive from the 28th year of their miniſtry to the cloſe of life, it is manifeſt that, during this interval, there ought to be a total exemption from the payment of the rate, as far as the above ſurplus of 1473—435 = L. 1038 will admit; ſeeing theſe contributors have paid, at a medium, the ſum of L. 324 more than the value of the annuity. Therefore,

I call upon you, Mr Preſes, I call upon the church in general, ſeeing you have it now in your power, to do an act of juſtice to our aged brethren, who have borne and ſtill will bear the burden in the heat of the day, by ſuſpending the payment of the rates after a certain age, or rather, by exempting aged contributors after a certain number of annual payments, including the marriage-tax, be that number what it will; which number, could I depend upon the only table of obſervations which approaches near to the better ſort of lives in Scotland, might be 30 *, including the marriage-tax; upon the ſuppoſition that each contributor thus exempted was obliged to pay, in name of vaſſalage, 5 per cent. or 1 s. upon the pound,

* It is required, to find, if L. 938 of annual ſurplus will admit of a ſuſpenſion of the rates beyond 30 annual payments, including the marriage-tax. Suppoſing 30 the mean age of admiſſion, there are alive, by the Fife table of obſervations, at the age of 30 years, 358—2=356, and at 60 years of age, 211—4.5=206.5, whoſe expectation of life is 12.98; and there are admitted into the church and univerſities 29.455 contributors yearly. Hence,
356 : 206.5 : : 29.455 : 17.08, and 12.98×17.08=222, alive at 60 and upwards. Hence, 222 × 4.25 = L. 943.5, the ſum remitted to theſe contributors.

of the annuity correſponding to the rate to which he was ſubjected, and that L. 100 of ſurplus was reſerved, in order to co-operate with the ſavings upon the rates and annuities, till the exemptees and annuitants arrived at a maximum, in forming an additional fund of L. 9000, to diſcharge the annuities of thoſe 18 widows who are ſtill to be provided for, as appears from the following

TABLE.

Age.	Rate.	Living.		Expect.		Total alive.
30	0	29.455	×	32.66	=	962.
42	12	25.85	×	24.29	=	628.
58	28	18.45	×	14.00	=	258.
59	29	17.79	×	13.50	=	240.
60	30	17.08	×	12.98	=	222.
61	31	16.34	×	12.53	=	205.
62	32	15.60	×	12.10	=	189.
63	33	14.85	×	11.66	=	173.

If the capital be allowed to riſe to L. 109,000, as ſuggeſted above, by the accumulation of L. 150 per annum, without intereſt, during the ſpace of 60 years, when the annuitants will arrive at a maximum: in this caſe, the above ſurplus of L. 1038, appropriated to contributors, with a tax of 1 s. per pound of the annuity, will reduce the years of exemption to 28, when a contributor hath paid the full value of the augmented annuity, and when the fourth part of thoſe alive at one

one time in the church will be exempted annually. Moreover,

If the annual ſurplus ſhall happen to riſe conſiderably above L. 1000, both plans of augmentation and exemption may be uſed to advantage; but ſhould it not exceed or fall below that ſum, the plan of exemption ought alone to be employed.——Now, that ſuch a plan ought to be adopted, in preference to all others, will appear,

1ſt, From its being founded upon a principle of juſtice. In ſuch a Widows Scheme as this, we acknowledge that charity, mercy, or compaſſion for the indigent, are the leading principles: therefore, as long as it is neceſſary for the exiſtence of the ſcheme, rates ought to be paid till the cloſe of life; but when this hardſhip, as in the preſent caſe, can be removed, and juſtice mixed with mercy, it ought to be done. With what confidence can we preſume to diminiſh the rates in general, or augment the annuities, whilſt we leave this burden upon aged contributors unremoved? A grievance which, remaining unredreſſed, muſt blaſt the reputation of every other plan, at leaſt in the opinion of every diſcerning perſon. Beſides, every one who is exempted after 30 annual payments of the rate, will have contributed a ſum, viz. L. 295, whoſe intereſt, at 4 per cent. is within a trifle of L. 12 per annum; whilſt the other contributors, from 30 years and under, will have only paid, each of them, at a medium, a ſum, viz. L. 124, whoſe intereſt, together with the annual rate, does not exceed L. 10.2 per annum: ſo that the former benefit the ſcheme as much after an exemption, as the latter do with the payment of the rate.

2d, From its poſſeſſing the beneficent properties of all other plans. It prevents the accumulation of the capital beyond a limited ſum; it augments the annuities with L. 435, and diminiſhes the rates to the extent of L. 1038 per annum, and that too moſt effectually where there is moſt occaſion for a diminution; and as a penny ſaved is ſo much gained, it may be ſaid to augment the annuity, in the caſe of exemptees, by the value of the rate during the life of the contributor. Moreover, was it thought adviſeable to credit the truſtees with the ſmall diſcretionary power of augmenting or diminiſhing the years of exemption, according as it ſhould be found in fact, that the number of annuitants or of exemptees exceeded or fell ſhort of the calculation, or that the intereſt of money was below $4\frac{1}{2}$ per cent.; this plan might keep the affairs of the ſcheme upon ſuch an even footing, as to prevent all future applications to parliament for redreſs of grievances, which cannot fail of being both expenſive and troubleſome; whilſt the loſſes ſuſtained by bad debts or otherwiſe are effectually guarded againſt by the law now in force.

3d, From the ſimplicity of its conſtruction and operation. Let any one attempt to diminiſh the rates or augment the annuities, he will find more difficulties than poſſibly he expected, and will be in danger of unhinging the whole frame of the ſcheme. But by this plan, the annuities are paid in guineas, in place of as many pounds Sterling, and the rates remain entire as from the commencement of the fund, which ſtill enjoys the full benefit of deductions and of half-years rates out

of

of anns and vacant ſtipends; ſo that the affairs of the ſcheme, in general, proceed in their uſual courſe, which cannot fail of being a great advantage, in point of eaſe, both to the truſtees and general collectors. The only calculation neceſſary will be to determine the years of exemption; for which purpoſe had we a table of obſervations adapted to the lives of miniſters, this might be done with accuracy; if not it may be diſcovered from fact, with this caution, that the number of exemptees will not arrive at a maximum till about the 85th year of the ſcheme. But let none object to this plan,

1*ſt*, That I am in danger of loſing the concurrence of the younger members of the church, who form the greater part of the ſociety. No; I will caſt myſelf upon their generoſity: Dr Webſter did ſo at the commencement of the ſcheme, and was not diſappointed. Had it not been for their diſintereſted conduct, the ſcheme would never have taken place; nor can I ſuppoſe that, in the courſe of half a century, this public ſpirit ſhould be degenerated in the younger members of the ſociety. But I aſk leſs than the Doctor did; I only aſk an act of juſtice towards their aged brethren, whilſt the youngeſt in the church have at leaſt a chance of reaping the ſame benefit with the oldeſt, and many may there be who fulfil the 30th year of their miniſtry, before which they have no cauſe to complain, not having diſcharged the full value of the annuity. Nor,

2*d*, That this plan comes from an obſcure corner of the church, and from an inconſiderable hand.

hand. No; However inconſiderable the author of this diſſertation may be in point of mental abilities, he is yet one who hath been deeply engaged in the ſcience of annuities of all deſcriptions, and in particular hath given ſuch an illuſtration of this very ſcheme; as he truſts will ſtand the teſt of the moſt impartial examination. But however this be, he is one who will yield to none in regard for his country, and in his good wiſhes for the proſperity of the Church of Scotland; in evidence of which, he hath taken this method of being ſerviceable to the church, in the reſolution of a queſtion which may come to be determined, when poſſibly this very hand is laid in the duſt.

DAVID WILKIE.

CULTS,
Oct. 18. 1792.

JUL 12 1920